Review of two problems

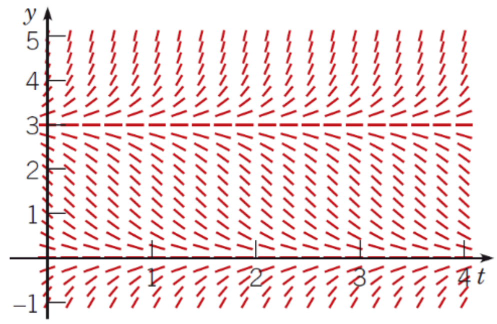


FIGURE 1. Slope Field

11. Among the differential equations below, which one has a slope field which is similar to the one shown in Figure 1?

A. $y' = y(3 - y)$

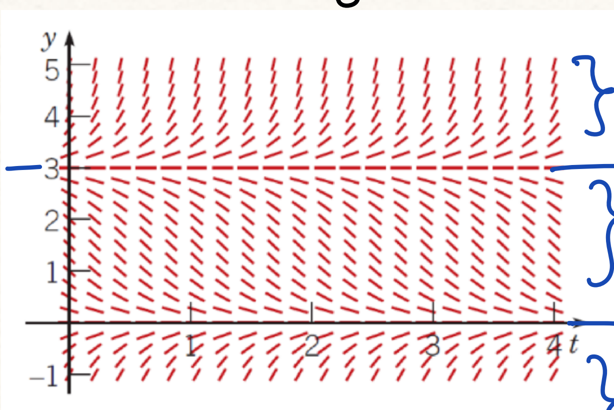
B. $y' = y - 3$

C. $y' = y(3 + y)$

D. $y' = y(y - 3)$

E. $y' = y + 3$

This looks like things about stable and unstable points (which it can be), but we can solve this by just inspecting the slope field and not referencing stable/unstable.



} function increasing ($y' > 0$)

3 – function flat ($y' = 0$)

} function decreasing ($y' < 0$)

0 function flat ($y' = 0$)

} function increasing ($y' > 0$).

Based on what we have here, the increasing/decreasing description of the function should look something like

For instance then, when $y=2$ we should have $y'<0$, when $y=0$ we should have $y'=0$, and so on. Specifically

$$y' \begin{cases} + & y>3 \\ 0 & y=3 \\ - & 0<y<3 \\ 0 & y=0 \\ + & y<0 \end{cases}$$

Now we just check which choice makes this work.

First only choices A and D have $y'=0$ when $y=0$ and $y=3$.

Choice A: $y' = y(3-y)$.

If we take $y=1$ though, we get $y' = 1(3-1) = 2 > 0$, which doesn't fit our descriptron above.

Thus, **choice D** is all that remains, and we could easily verify that it matches our description.

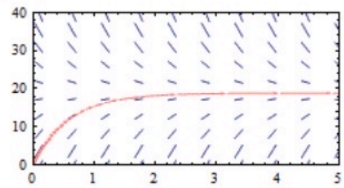
You bail out of a helicopter and pull the ripcord of your parachute. Now the air resistance proportionality constant is k = 2 , so your downward velocity satisfies the initial value problem below, where v is measured in ft/s and t in seconds. In order to investigate your chances of survival, construct a slope field for this differential equation and sketch the appropriate solution curve. What will your limiting velocity be?

Assume acceleration due to gravity is 32 ft/sec^2

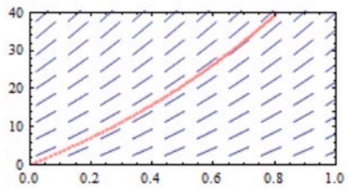
Construct a slope field for this differential equation and sketch the appropriate solution curve. Choose the correct graph below.

Because I changed k=2, the graphs no longer match the numbers.

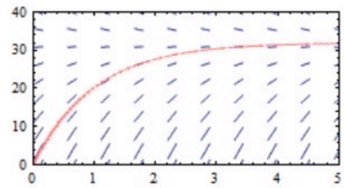
A.



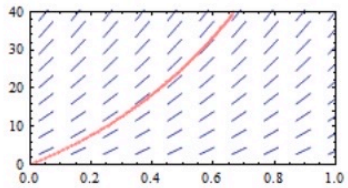
B.



C.

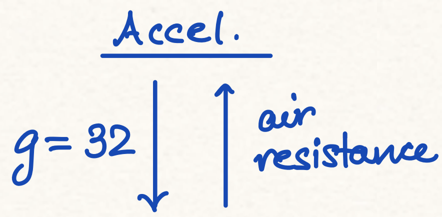


D.



Because we have gravity and air resistance, our acceleration is $\frac{dv}{dt} = 32 - 2v$

(32 = g, 2v = air resistance)



where air resistance is $-2v$ because it points opposite to the way we are falling.

★ say velocity down is positive for simplicity

Then accel. $= \frac{dv}{dt} = 2(16 - v)$.

What does the slope field for v look like?

?

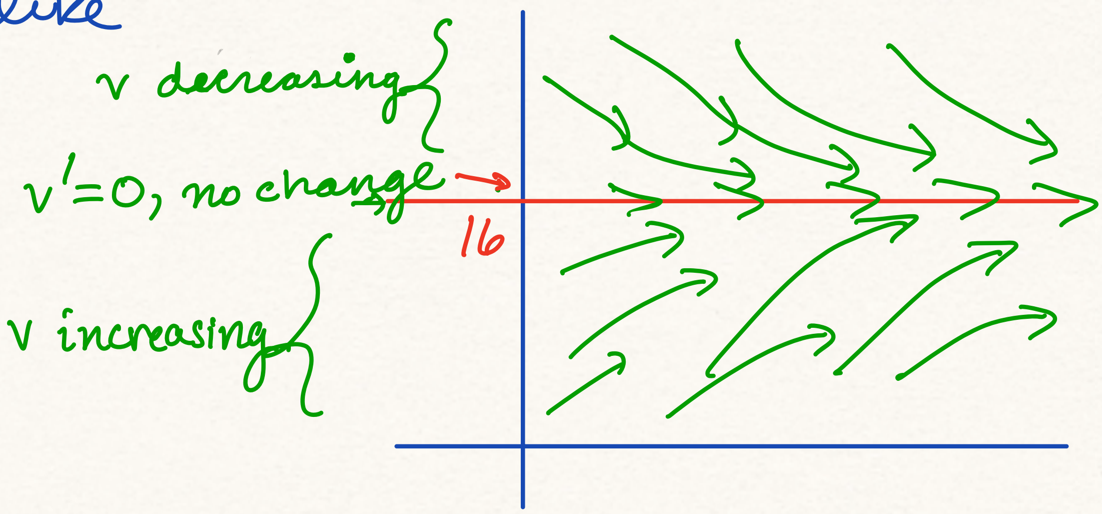
well, as $v' = 2(16-v)$, we see that

$v' = 0$ at $v = 16$,

if $v < 16$ then $v' = 2(16-v) > 0$

if $v > 16$ then $v' = 2(16-v) < 0$.

So it's reasonable a slope field might look like



So we conclude that the terminal velocity (place where $v' = 0$) is at $v = 16$ (ft/sec)

The graph that is closest to this is choice A