Practice problems for Exam 1

1.

Find the general solution of the differential equation

$$y' + x^{-1}y = xy^2.$$

2.

Consider the equation

$$(xy^2 + kx^2y)dx + (x^3 + yx^2)dy = 0.$$

(a) Find the value of the constant k for which the equation is exact.
(b) Solve the equation using the value of k found in part (a).

3.

Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}.$$

4.

Find the solution of the initial value problem

$$y' + \frac{2}{x}y = 4x, \qquad y(1) = 2.$$

5.

Given the matrix

$$A = \begin{pmatrix} -1 & 3 & 0 \\ 2 & -1 & 1 \\ -\frac{10}{3} & 0 & -2 \end{pmatrix}$$

find the solutions to the homogeneous system $A\mathrm{x} = \mathbf{0}$.

6.

Let $A = \begin{pmatrix} k & 1 \\ 5 & k \end{pmatrix}$ and $\mathrm{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. For which values of k does the linear system $A\mathrm{x} = \mathrm{b}$ have a unique solution?

7.

If $A = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & -1 & 0 & -3 \\ 0 & 1 & -1 & 2 \\ -1 & 1 & 2 & 0 \end{pmatrix}$, and we denote $A^{-1} = [b_{ij}]$, find b_{32}.

8.

Determine which one(s) of the following initial value problems have a unique solution, according to the Existence and Uniqueness of Solution Theorem:

$\frac{dy}{dx} = y^4 - x^4$, $y(0) = 7$;

$y\frac{dy}{dx} = x$, $y(1) = 0$.

9.

The vectors $(1, 2, 3, a)$, $(1, 2, 3, 4)$, $(2a, 3, -1, 3)$, and $(1, 1, 2, 2)$ are linearly independent

A. when $a = 4$ and $a = -2$

B. when $a = -2$

C. for all $a \neq 4$

D. for all values of a

E. when $a \neq 4$ and $a \neq -2$

Correct Answer is **E**

10.

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(1, 0, 0) = (1, 0, 1)$, $T(0, 1, 0) = (2, 1, 0)$, and $T(0, 0, 1) = (0, -1, 2)$. If

$$\mathbf{v}_1 = (0, 0, 0), \quad \mathbf{v}_2 = (0, 0, 3), \quad \mathbf{v}_3 = (4, 1, 2),$$

which of the following vectors are in $\text{Rng}(T)$?

A. $\mathbf{v}_1$ and $\mathbf{v}_2$ only

B. $\mathbf{v}_1$ and $\mathbf{v}_3$ only

C. $\mathbf{v}_2$ and $\mathbf{v}_3$ only

D. $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$

E. None of the above

Correct Answer is **B**