Practice Problems for Exam 2

1.

Find the rank of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ 3 & 0 & -3 \\ 2 & 0 & 1 \end{pmatrix}.$$

2.

If

$$det\ \mathbf{A} = \begin{pmatrix} 0 & a & 0 \\ 1 & 2 & 3 \\ 4 & 3 & 6 \end{pmatrix} = 18,$$

a) Find a;
b) Compute $\det \mathbf{A}^T$.

3.

Consider the three vectors in $\mathbb{R}^3$

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}.$$

Prove that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ span $\mathbb{R}^3$.

4.

Determine which of the following subsets S is a subspace of the vector space $\mathbf{V}$. Provide motivation for your answers.

(i) $\mathbf{V} = \mathbb{R}^3$, $S = \{(x, y, z) \in \mathbb{R}^3 \mid 2(x-1) - 3(y+1) + (z+7) = 2\}$.

(ii) $\mathbf{V} = M_{2\times 2}(\mathbb{R})$, $S = \left\{ \mathbf{A} \in M_{2\times 2}(\mathbb{R}) \mid \mathbf{A} = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \right\}$.

(iii) $\mathbf{V} = C^2(I)$, where I is an interval of the line, $S = \{f \in C^2(I) \mid f''(x) + 4f'(x) - 3f(x) = 1\}$.

5.

The general solution of $y'' + 2y' + 5y = 0$ is $y =$

A. $e^{-x}(A\cos x + B\sin x)$ B. $Ae^{-2x} + Bxe^{-2x}$ C. $e^{-x}(A\cos 2x + B\sin 2x)$
D. $Ae^{3x} + Be^{2x}$ E. $e^{-x}(A\cos\sqrt{2}x + B\sin\sqrt{2}x)$

6.

Which of the following are vector spaces?

i) the set of all 2×2 non-singular matrices
ii) the set of all continuous functions with $f(a) = f(a + 2\pi)$
iii) the set of all vectors of the form $(r + s, r, r - s)$, r, s real

A. (i) and (ii) B. (i) and (iii) C. (ii) and (iii) D. (i), (ii) and (iv) E. only (iii)

7.

Find the value of k for which the following 4 vectors fail to span $\mathbb{R}^3$;

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}. \quad v_3 = \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 6 \\ k \end{bmatrix}$$

A. $k = 0$ B. $k = 1$ C. $k = 2$ D. $k = 3$ E. $k = 4$

8.

The product of the eigenvalues of the matrix $M = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ is

A. 2 B. 3 C. 4 D. 5 E. 6

9.

If

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } B = A^{-1} \text{ then } b_{31} =$$

A. 0 B. 1 C. -1 D. 3 E. -3

10.

The matrix $A = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$ has an eigenvalue $-2 + i$. An eigenvector of A is

A. $(1 + i, 2)$ B. $(1 - i, 2)$ C. $(1 + i, -2)$ D. $(2 + i, 1)$ E. $(2 - i, 1)$