Student: ______________________ **Date:** ______________________	**Instructor:** DONATELLA DANIELLI, Jennifer Hobbs, Pavel Coupek **Course:** MA 262-525- Spring 2020	**Assignment:** Trial Midterm 2

1. Find the dimensions of the null space and the column space of the given matrix.

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 & 0 & 5 & -4 \\ 0 & 0 & 1 & -6 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A 4×7 7 unknowns

Nul space $= \{\bar{x}' \in \mathbb{R}^7 \mid A\bar{x}' = \bar{0}'\}$

3 equs. $7-3=4$

4 free parameters

dim Nul (A) = 4

dim Col (A) = 3

- A. dim Nul A = 2, dim Col A = 5
- B. dim Nul A = 3, dim Col A = 4
- C. dim Nul A = 4, dim Col A = 3 ✗
- D. dim Nul A = 5, dim Col A = 2

2. Let W be the union of the first and third quadrants in the xy-plane. That is, let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$. Complete parts a and b below.

a. If **u** is in W and c is any scalar, is c**u** in W? Why?

- A. If $\mathbf{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ is in W, then the vector $c\mathbf{u} = c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$ is in W because $cxy \geq 0$ since $xy \geq 0$.
- B. If $\mathbf{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ is in W, then the vector $c\mathbf{u} = c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$ is not in W because $cxy \leq 0$ in some cases.
- C. ✗ If $\mathbf{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ is in W, then the vector $c\mathbf{u} = c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$ is in W because $(cx)(cy) = c^2(xy) \geq 0$ since $xy \geq 0$.

b. Find specific vectors **u** and **v** in W such that **u** + **v** is not in W. This is enough to show that W is *not* a vector space.

Two vectors in W, **u** and **v**, for which **u** + **v** is not in W are ___________. $\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

(Use a comma to separate answers as needed.)

$\bar{u}' \in W$ $\quad c \in \mathbb{R}$

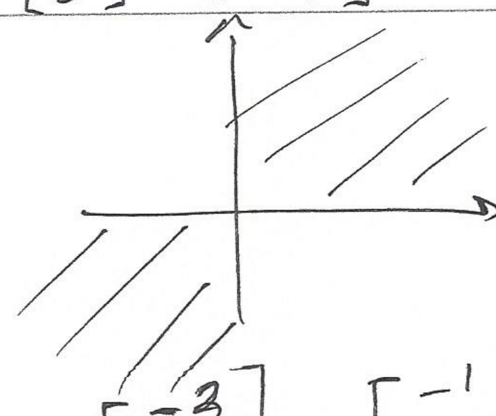
$c\bar{u}' \in W$? yes

$c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$

$(cx)(cy) \overset{?}{\geq} 0$

$\quad ||$

$c^2 xy \geq 0$ ✓



$\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

3. Let A and B be 3×3 matrices, with $\det A = -2$ and $\det B = 4$. Use properties of determinants to complete parts (a) through (e) below.

a. Compute det AB.

det AB = -8 (Type an integer or a fraction.)

$\det(AB) = \det(A) \det(B) = -2 \cdot 4 = -8$

b. Compute det 5A.

det 5A = -250 (Type an integer or a fraction.)

$A = 3 \times 3 \quad \det(5A) = 5^3 \cdot \det(A) = 125 \cdot -2 = -250$

c. Compute $\det B^T$.

$\det B^T =$ 4 (Type an integer or a fraction.)

$\det(B^T) = \det(B)$

d. Compute $\det A^{-1}$.

$\det A^{-1} = \frac{1}{-2} = -\frac{1}{2}$ (Type an integer or a simplified fraction.)

$\det(A^{-1}) = \frac{1}{\det(A)}$

$1 = \det(I_3) = \det(A)\det(A^{-1})$

e. Compute $\det A^3$.

$\det A^3 = -2^3 = -8$ (Type an integer or a fraction.)

$\det(A^3) = \det(A) \cdot \det(A) \cdot \det(A) = \det(A)^3$

4. Combine the methods of row reduction and cofactor expansion to compute the determinant.

$$\begin{vmatrix} -1 & 4 & 9 & 0 \\ 4 & 3 & 5 & 0 \\ 4 & 4 & 6 & 4 \\ 4 & 2 & 4 & 2 \end{vmatrix}$$

The determinant is __________.
(Simplify your answer.)

5. Let the matrix below act on $\mathbb{C}^2$. Find the eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$.

$$\begin{bmatrix} 5 & -2 \\ 2 & 5 \end{bmatrix}$$

The eigenvalues of $\begin{bmatrix} 5 & -2 \\ 2 & 5 \end{bmatrix}$ are __________.
(Type an exact answer, using radicals and i as needed. Use a comma to separate answers as needed.)

A basis for the eigenspace corresponding to the eigenvalue $a + bi$, where $b > 0$, is __________.
(Type an exact answer, using radicals and i as needed.)

A basis for the eigenspace corresponding to the eigenvalue $a - bi$ where $b > 0$, is __________.
(Type an exact answer, using radicals and i as needed.)

#4

$$\begin{vmatrix} -1 & 4 & 9 & 0 \\ 4 & 3 & 5 & 0 \\ 4 & 4 & 6 & 4 \\ 4 & 2 & 4 & 2 \end{vmatrix} = - \begin{array}{c} -4 \\ \\ \\ \\ \end{array}\begin{vmatrix} 1 & -4 & -9 & 0 \\ 4 & 3 & 5 & 0 \\ 4 & 4 & 6 & 4 \\ 4 & 2 & 4 & 2 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & -4 & -9 & 0 \\ 0 & 19 & 41 & 0 \\ 0 & 20 & 42 & 4 \\ 0 & 18 & 40 & 2 \end{vmatrix} = - \begin{vmatrix} 19 & 41 & 0 \\ 20 & 42 & 4 \\ 18 & 40 & 2 \end{vmatrix}$$

$$= - \begin{array}{c} \cancel{-20}\ -18 \\ \\ \\ \end{array}\begin{vmatrix} 1 & 1 & -2 \\ 20 & 42 & 4 \\ 18 & 40 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & -2 \\ 0 & 22 & 44 \\ 0 & 22 & 38 \end{vmatrix}$$

$$= - \begin{vmatrix} 22 & 44 \\ 22 & 38 \end{vmatrix}$$

$$= -(22 \cdot 38 - 22 \cdot 44)$$

$$= -22(38 - 44) = -22 \cdot (-6)$$

$$= 132 \checkmark$$

#5

$$\begin{bmatrix} 5 & -2 \\ 2 & 5 \end{bmatrix} = A$$

Evalues

$$A - \lambda I = \begin{bmatrix} 5-\lambda & -2 \\ 2 & 5-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (5-\lambda)^2 + 4 = 0$$

$$(5-\lambda)^2 = -4$$

$$5 - \lambda = \pm 2i$$

$$\lambda = 5 \pm 2i$$

$$\lambda_1 = 5 + 2i$$

$$A - (5+2i)I = \begin{bmatrix} \cancel{5}-\cancel{5}-2i & -2 \\ 2 & \cancel{5}-\cancel{5}-2i \end{bmatrix}$$

$$= \begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix} \sim^{-i} \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -i \\ 0 & \cancel{1-1}\,0 \end{bmatrix}$$

$$v_1 - i v_2 = 0 \qquad v_1 = i v_2$$

$$v_2 = 1 \qquad v_1 = i$$

$$\begin{bmatrix} i \\ 1 \end{bmatrix}$$

#5 cont.

$\lambda_2 = 5 - 2i$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & -2 \\ 2 & 5-\lambda \end{bmatrix}$$

$$A - (5-2i) I = \begin{bmatrix} \cancel{5} - \cancel{5} + 2i & -2 \\ 2 & 2i \end{bmatrix}$$

$$\sim \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \sim {}^{-i} \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & i \\ 0 & \cancel{-1+1}\ 0 \end{bmatrix} \qquad v_1 + i v_2 = 0$$

$$v_1 = -i v_2$$

$v_2 = 1 \qquad v_1 = -i$

$$\begin{bmatrix} -i \\ 1 \end{bmatrix}$$

6. Is $\begin{bmatrix} 2+\sqrt{6} \\ -2 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}$? If so, find the eigenvalue.

Select the correct choice below and, if necessary, fill in the answer box within your choice.

○ A. Yes, $\begin{bmatrix} 2+\sqrt{6} \\ -2 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}$. The eigenvalue is $\lambda =$ ________.
(Type an exact answer, using radicals as needed.)

○ B. No, $\begin{bmatrix} 2+\sqrt{6} \\ -2 \end{bmatrix}$ is not an eigenvector of $\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}$.

7. Use Cramer's rule to compute the solution of the system.

$$\begin{aligned} x_1 + x_2 \quad &= 2 \\ -3x_1 \quad + 3x_3 &= 0 \\ x_2 - 3x_3 &= 1 \end{aligned}$$

$x_1 =$ ________; $x_2 =$ ________; $x_3 =$ ________
(Type integers or simplified fractions.)

8. Suppose $\mathbb{R}^4 = \text{Span}\{\mathbf{v}_1,\ldots,\mathbf{v}_4\}$. Explain why $\{\mathbf{v}_1,\ldots,\mathbf{v}_4\}$ is a basis for $\mathbb{R}^4$.

Complete the explanation below.

Let $A = [\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3\ \mathbf{v}_4]$. Note that A is a (1) ________ matrix and its columns span (2) ________ Thus, by the (3) ________ the columns (4) ________ Therefore, the columns of A are a basis for $\mathbb{R}^4$ because of the (5) ________

(1) ☒ 4×4
○ 1×4

(2) ○ $\mathbb{R}$.
☒ $\mathbb{R}^4$.

(3) ☒ Invertible Matrix Theorem,
○ Basis Theorem,
○ definition of a basis,
○ definition of linear independence,
○ Rank Theorem,
○ Spanning Set Theorem,

(4) ○ are linearly dependent.
○ are pivot columns.
○ span $\mathbb{R}^3$.
☒ are linearly independent.

(5) ○ Invertible Matrix Theorem.
○ Basis Theorem.
☒ definition of a basis.
○ definition of a spanning set.
○ Spanning Set Theorem.
○ Rank Theorem.

Basis : i) LI
ii) span.

#6 $\quad A = \begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 2+\sqrt{6} \\ -2 \end{bmatrix}$

$(A - \lambda I)\vec{v} = \vec{0}$?

$$\begin{bmatrix} 5-\lambda & -1 \\ -2 & 1-\lambda \end{bmatrix} \begin{bmatrix} 2+\sqrt{6} \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 5\sqrt{6} - 2\lambda - \sqrt{6}\lambda + 2 \\ -4 - 2\sqrt{6} - 2 + 2\lambda \end{bmatrix}$$

$$= \begin{bmatrix} 12 + \sqrt{6}(5-\lambda) - 2\lambda \\ -6 - 2\sqrt{6} + 2\lambda \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$12 + \sqrt{6}(5-\lambda) - 2\lambda = 0$$
$$-6 - 2\sqrt{6} + 2\lambda = 0$$

$$2\lambda = 6 + 2\sqrt{6} \qquad \lambda = 3 + \sqrt{6}$$

$$12 + \sqrt{6}(5 - 3 - \sqrt{6}) - 2(3+\sqrt{6}) \stackrel{?}{=} 0$$

$$12 + 2\sqrt{6} - 6 - 6 - 2\sqrt{6} = 0$$

#7

$$\begin{cases} x_1 + x_2 = 2 \\ -3x_1 + 3x_3 = 0 \\ x_2 - 3x_3 = 1 \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -3 & 0 & 3 \\ 0 & 1 & -3 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 = \frac{\det(B_1)}{\det(A)} \qquad B_1 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 3 \\ 1 & 1 & -3 \end{bmatrix}$$

$$\det(A) = 1 \begin{vmatrix} 0 & 3 \\ 1 & -3 \end{vmatrix} - 1 \begin{vmatrix} -3 & 3 \\ 0 & -3 \end{vmatrix}$$

$$= -3 - 9 = -12 \neq 0$$

$$\det(B_1) = -3 \begin{vmatrix} +2 & 1 \\ 1 & 1 \end{vmatrix} =$$

$$-3 \cdot 1 = -3$$

$$x_1 = \frac{\det(B_1)}{\det(A)} = \frac{-3}{-12} = 1/4$$

$$x_2 = \frac{\det(B_2)}{\det(A)}$$

$$B_2 = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 0 & 3 \\ 0 & 1 & -3 \end{bmatrix}$$

#7 cont.

$$\det(B_2) = \begin{vmatrix} 1 & 2 & 0 \\ -3 & 0 & 3 \\ 0 & 1 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 3 \\ 1 & -3 \end{vmatrix} - 2 \begin{vmatrix} -3 & 3 \\ 0 & -3 \end{vmatrix}$$

$$= -3 - 2 \cdot 9 = -21$$

$$x_2 = \frac{\det(B_2)}{\det(A)} = \frac{+21}{+12} = +\frac{7}{4}.$$

$$x_3 = \frac{\det(B_3)}{\det(A)}$$

$$B_3 = \begin{bmatrix} 1 & 1 & 2 \\ -3 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det(B_3) = -(-3) \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} =$$

$$3(-1) = -3$$

$$x_3 = {-3}/{-12} = 1/4.$$

9. Determine if the given set is a subspace of $\mathbb{P}_8$. Justify your answer.

The set of all polynomials of the form $\mathbf{p}(t) = at^8$, where a is in $\mathbb{R}$.

Choose the correct answer below.

- **A.** The set is not a subspace of $\mathbb{P}_8$. The set does not contain the zero vector of $\mathbb{P}_8$.
- **B.** The set is a subspace of $\mathbb{P}_8$. The set contains the zero vector of $\mathbb{P}_8$, the set is closed under vector addition, and the set is closed under multiplication by scalars.
- **C.** The set is a subspace of $\mathbb{P}_8$. The set contains the zero vector of $\mathbb{P}_8$, the set is closed under vector addition, and the set is closed under multiplication on the left by $m \times 8$ matrices where m is any positive integer.
- **D.** The set is not a subspace of $\mathbb{P}_8$. The set is not closed under multiplication by scalars when the scalar is not an integer.

10. If the null space of a 6×8 matrix is 5-dimensional, find rank A, dim Row A, and dim Col A.

- **A.** rank A = 3, dim Row A = 5, dim Col A = 5
- **B.** rank A = 3, dim Row A = 3, dim Col A = 5
- **C.** rank A = 1, dim Row A = 1, dim Col A = 1
- **D.** rank A = 3, dim Row A = 3, dim Col A = 3

$A\bar{x}' = 0$ 5 free parameters
8 unknowns
$8 - 5 = 3$ # of valid eqns.
rank (A) = 3 = dim Row (A)
= dim Col(A)

11. Let $A = \begin{bmatrix} -16 & -10 & -22 \\ 357 & 200 & 442 \\ 100 & 55 & 124 \end{bmatrix}$. Find the second and third columns of A^{-1} without computing the first column.

$$\begin{bmatrix} -8 & -5 & -11 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

How can the second and third columns of A^{-1} be found without computing the first column?

- **A.** Solve the equation $A\mathbf{e}_2 = \mathbf{b}$ for $\mathbf{e}_2$, where $\mathbf{e}_2$ is the second column of I_3 and $\mathbf{b}$ is the second column of A^{-1}. Then similarly solve the equation $A\mathbf{e}_3 = \mathbf{b}$ for $\mathbf{e}_3$.
- **B.** Row reduce the augmented matrix $[A \; I_3]$.
- **C.** Row reduce the augmented matrix $[A \; \mathbf{e}_2 \; \mathbf{e}_3]$, where $\mathbf{e}_2$ and $\mathbf{e}_3$ are the second and third columns of I_3.
- **D.** Row reduce the augmented matrix $\begin{bmatrix} A \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix}$, where $\mathbf{e}_2$ and $\mathbf{e}_3$ are the second and third columns of I_3.

The second column of A^{-1} is ____________.
(Type an integer or decimal for each matrix element. Round to four decimal places as needed.)

The third column of A^{-1} is ____________.
(Type an integer or decimal for each matrix element. Round to four decimal places as needed.)

#9 $p(t) = at^8 \qquad a \in \mathbb{R}$

① $p(t) \equiv 0 \qquad a = 0 \Rightarrow p(t) \equiv 0$ ✓

② Closure under addition ✓

$p(t) = at^8 \qquad q(t) = bt^8$

$(p+q)(t) = at^8 + bt^8 = \underbrace{(a+b)}_{\in \mathbb{R}}t^8$

③ Closure under scalar multiplication ✓

$c \in \mathbb{R}$

$(cp)(t) = c\,p(t) = \underbrace{ca}_{\in \mathbb{R}}t^8$

#11

$$\begin{bmatrix} A & I_n \end{bmatrix} \qquad \begin{bmatrix} A & \cancel{\vec{e}_1} & \vec{e}_2 & \vec{e}_3 \end{bmatrix}$$

$$\downarrow \text{RREF}$$

$$\begin{bmatrix} I_n & A^{-1} \end{bmatrix}$$

$$A = \begin{bmatrix} -8 & -5 & -11 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -8 & -5 & -11 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$-2\,\cancel{8} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ -8 & -5 & -11 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & -5 & -3 & 8 & 0 \\ 0 & 1 & -2 & -2 & 1 \end{bmatrix}$$

$$5 \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -2 & -2 & 1 \\ 0 & -5 & -3 & 8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -2 & -2 & 1 \\ 0 & 0 & -13 & -2 & 5 \end{bmatrix}$$

#11 cont.

$$\begin{array}{c} \\ \\ 2 \end{array}\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -2 & -2 & 1 \\ 0 & 0 & 1 & \frac{2}{13} & -\frac{5}{13} \end{bmatrix}$$

$$\left[\begin{array}{ccccc} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_3} & \begin{matrix} \frac{11}{13} \\ -\frac{22}{13} \\ \frac{2}{13} \end{matrix} & \begin{matrix} +\frac{5}{13} \\ \frac{3}{13} \\ -\frac{5}{13} \end{matrix} \end{array}\right]$$

$\downarrow$ 2nd col. of A^{-1}

$\downarrow$ 3rd col. of A^{-1}

Practice Problems for Exam 2

$-\frac{2}{3}$ $\begin{bmatrix} 1 & -1 & -2 \\ 3 & 0 & -3 \\ 2 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 3 \\ 0 & 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & -2 \\ 0 & 2 & 5 \end{bmatrix}$

1.

Find the rank of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ 3 & 0 & -3 \\ 2 & 0 & 1 \end{pmatrix}.$$

$\text{rank}(A) = 3$

2.

If

$$det\ \mathbf{A} = \begin{pmatrix} 0 & a & 0 \\ 1 & 2 & 3 \\ 4 & 3 & 6 \end{pmatrix} = 18,$$

a) Find a;
b) Compute $\det \mathbf{A}^T$. $= \det(A) = 18$

3.

Consider the three vectors in $\mathbb{R}^3$

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}.$$

Prove that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ span $\mathbb{R}^3$.

4.

Determine which of the following subsets S is a subspace of the vector space $\mathbf{V}$. Provide motivation for your answers.

(i) $\mathbf{V} = \mathbb{R}^3$, $S = \{(x, y, z) \in \mathbb{R}^3 \mid 2(x-1) - 3(y+1) + (z+7) = 2\}$.

(ii) $\mathbf{V} = M_{2\times 2}(\mathbb{R})$, $S = \left\{ \mathbf{A} \in M_{2\times 2}(\mathbb{R}) \mid \mathbf{A} = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \right\}$.

(iii) $\mathbf{V} = C^2(I)$, where I is an interval of the line, $S = \{f \in C^2(I) \mid f''(x) + 4f'(x) - 3f(x) = 1\}$.

(iii) (1) 0 property

$f(x) \equiv 0$ $\quad f \in S$?

$f' \equiv 0$ $\quad 0 \cancel{=} 1$

$f'' = 0$

S not subspace.

#3 $A = [\bar{v}_1' \quad \bar{v}_2' \quad \bar{v}_3']$

Reduce REF

$$\text{rank}(A) = \begin{cases} 3 & \text{span} \\ \neq 3 & \text{do not span} \end{cases}$$

#4 (i) $2(x-1) - 3(y+1) + (z+7) = 2$

$2x - 2 - 3y - 3 + z + 7 = 2$

$\rightarrow 2x - 3y + z = 0$

(1) 0 belongs

$x = y = z = 0 \longrightarrow 0 = 0$ ✓

$(x, y, z) = (0, 0, 0)$ belongs ✓

(2) Closure under addition ✓

$(x, y, z) : 2x - 3y + z = 0 \leftarrow$

$(u, v, w) : 2u - 3v + w = 0$

$(x, y, z) + (u, v, w) = (x+u, y+v, z+w)$

$2(x+u) - 3(y+v) + (z+w) \stackrel{?}{=} 0$

$\underbrace{2x - 3y + z}_{=0} + \underbrace{2u - 3v + w}_{=0} = 0$ ✓

(3) Closure under scalar mult. ✓

$c(x, y, z) = (cx, cy, cz)$

$2\cancel{c}x - 3\cancel{c}y + \cancel{c}z = 0$ ✓

#4(ii)

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

(1) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in S$?

$a = b = c = 0$ ✓

(2) Addition ✓

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \quad B = \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$$

$$A + B = \begin{bmatrix} a+x & b+y \\ 0 & c+z \end{bmatrix}$$

(3) Scalar mult. ✓

$k \in \mathbb{R}$

$$kA = \begin{bmatrix} ka & kb \\ 0 & kc \end{bmatrix}$$ ✓

S subspace

(iii)