Math 351, Midterm 1

February 19, 2015

Name: ______________________________

This exam consists of six pages including this front page.

Rules

1. You have one hour to complete the exam.
2. No calculators, books, or notes are allowed.
3. Show your work for every problem. Unjustified answers will receive no credit.
4. You may write on the front and back of each page. Extra paper is available.

Score		
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. Explain why each of the statements is either true or false. If false, please provide a counterexample.

(a) A nonempty subset of a linearly independent set is linearly independent.

(b) A linear system with more unknowns than equations must be consistent.

(c) Let A and B be matrices such that B is row equivalent to A. Then the linear system whose augmented matrix is B is equivalent to the linear system whose augmented matrix is A.

(d) A system of linear equations can have exactly two solutions.

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

(a) Find all vectors $B \in \mathbb{R}^3$ such that the equation $AX = B$ is consistent.

(b) Calculate the nullspace of A.

(c) Provide a linearly independent set of vectors which span the column space of A.

3. Consider the linear system

$$\begin{array}{rcrcrcrcl} 3w & + & 4x & + & 5y & + & 3z & = & 1 \\ w & + & x & + & y & + & z & = & 0 \\ w & + & 2x & + & 3y & + & z & = & 1 \\ 3w & + & 5x & + & 7y & + & 3z & = & 2 \end{array}$$

of four equations in four unknowns.

(a) Calculate the solution set of this linear system, clearly identifying the translation and spanning vectors.

(b) Calculate the rank of this linear system.

4. Let $S = \{X_1, X_2, X_3, X_4\}$ be the set of vectors

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad X_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

in $\mathbb{R}^4$.

(a) Show that S is linearly independent.

(b) Conclude that S is a basis for $\mathbb{R}^4$.

5. (a) Find all $b \in \mathbb{R}$ such that the line $y = ax + b$ is a subspace of $\mathbb{R}^2$.

(b) Show that the nullspace $\mathrm{Null}(A)$ of an $m \times n$ matrix A is a subspace of the vector space $\mathbb{R}^n$.

(c) Calculate the dimension of the vector space P_2 of polynomial functions $f(x) = a + bx + cx^2$ of degree less than or equal to two.