Math 351, Midterm 2

November 6, 2014

Name: ______________________________

This exam consists of six pages including this front page.

Rules

1. You have one hour to complete the exam.
2. No calculators, books, or notes are allowed.
3. Show your work for every problem. Unjustified answers will receive no credit.
4. You may write on the front and back of each page. Extra paper is available.

Score		
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. Explain why each of the statements is either true or false. If false, please provide a counterexample.

(a) If A and B are 2×2 matrices then $(AB)^2 = A^2B^2$.

(b) Suppose A is an $n \times n$ matrix satisfying the equation $A^2 - A + I = 0$. Then A is invertible.

(c) Suppose that A is an $m \times n$ matrix such that $AX = B$ has a solution for all $B \in \mathbb{R}^m$. Then for any $B' \in \mathbb{R}^n$, the solution to $A^t X = B'$, when it exists, is unique.

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 4 & 1 \end{bmatrix}.$$

(a) Calculated the LU factorization of A.

(b) Calculate the inverse of A.

3. Let $\mathcal{W}$ be the subspace of $M(2,2)$ consisting of those matrices of the form

$$\begin{bmatrix} a+b+3c & 2a-b \\ 0 & 2a+b+4c \end{bmatrix}$$

where a, b, and c range over all real numbers.

(a) Find a set of 2 x 2 matrices that span $\mathcal{W}$.

(b) Find a basis for $\mathcal{W}$.

(c) Calculate the dimension of $\mathcal{W}$.

4. (a) Determine whether the set of vectors

$$S = \left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$$

in $\mathbb{R}^3$ is linearly dependent or linearly independent. If S is linearly dependent, exhibit a dependency relation amongst the vectors in S.

(b) Let $T : \mathcal{V} \to \mathcal{W}$ be a linear transformation and let $\{X_1, X_2, X_3\}$ be a subset of $\mathcal{V}$. Show that if $\{T(X_1), T(X_2), T(X_3)\}$ is a linearly independent subset of $\mathcal{W}$ then $\{X_1, X_2, X_3\}$ is linearly independent.

5. Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be the transformation given by the formula

$$L([x, y, z]^t) = [x + y, y + z].$$

(a) Show that L is a linear transformation.

(b) Find the matrix A such that $L(X) = AX$ for all $X = [x, y, z]^t \in \mathbb{R}^3$.

(c) Find the matrix representation M of L with respect to the ordered bases $\mathcal{B} = \{[1, 0, 0]^t, [0, 1, 0]^t, [1, -1, 1]^t\}$ of $\mathbb{R}^3$ and $\overline{\mathcal{B}} = \{[1, 0]^t, [1, 1]^t\}$ of $\mathbb{R}^2$.