MA 351 Make Final

Last Name:____

First Name:____

No notes, calculators, or other electronic devices such as cell phones are allowed during the exam. Violation of this policy will result in an automatic 0 on the test. There should be nothing on your desk other than the test and something to write with. Please put all work and answers on either the test sheets or, if extra space is needed, on the attached scratch paper.

Justify all answers. A correct answer without supporting justification is worth NO credit!

(1) Use row reduction to find the inverse of the following matrix A. **Other techniques will not give credit.** Be sure to show enough steps so that I know that you know what you are doing.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 0 & 1 & 0 \end{bmatrix}$$

(2) Suppose that A is an $n \times n$ matrix such that $A^3 = \mathbf{0}$. Prove that $I + A$ is invertible and $(I + A)^{-1} = I - A + A^2$.

(3) Use the linearity properties to prove that the transformation $T : \mathbb{R}^3 \mapsto \mathbb{R}^2$ defined below is linear.

$$T([x, y, z]^t) = [x + z, y + 2x]^t.$$

(4) Let A be a 3×4 matrix having rank 3. Prove that there is a 4×3 matrix B such that $AB = I$.

(5) We use the ordered basis $\mathcal{B} = \{[1,1]^t, [0,2]^t\}$ to define coordinates for $\mathbb{R}^2$.

(a) Find the point matrix $P_{\mathcal{B}}$.

(b) Find the coordinate matrix $C_{\mathcal{B}}$.

(c) What point in $X \in \mathbb{R}^2$ has coordinate vector $X' = [3, 4]^t$?

(d) Find the coordinate vector X' for the point $X = [5, 1]^t$.

(6) Recall that $\mathcal{P}_2$ is the space of all polynomials of the form $f(x) = a + bx + cx^2$ where $a, b, c \in \mathbb{R}$. Compute the matrix M with respect to the standard ordered basis for $\mathcal{P}_2$ for the linear transformation $L : \mathcal{P}_2 \to \mathcal{P}_2$ where

$$L(f(x)) = f'(x) + f(x).$$

(7) Prove that if A is an $n \times n$ matrix for which there exists an $n \times n$ matrix B such that $BA = I$ then A is invertible and $B = A^{-1}$.

(8) Let $T : \mathcal{V} \to \mathcal{W}$ be a linear transformation between two vector spaces such that the nullspace of T is $\{\mathbf{0}\}$. Let $\{X_1, X_2, X_3\}$ be a set of three linearly independent elements in $\mathcal{V}$. Prove that then $\{T(X_1), T(X_2), T(X_3)\}$ is linearly independent.