Midterm Exam 3

Last Name:____

First Name:____

No notes, calculators, or other electronic devices such as cell phones are allowed during the exam. Violation of this policy will result in an automatic 0 on the test. There should be nothing on your desk other than the test and something to write with. Please put all work and answers on the test sheets. If extra space is needed, please the backs of the pages.

Justify all answers. A correct answer without supporting justification is worth NO credit!

(1) Given that $\det A = 10$ where

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

find $\det B$ where

$$B = \begin{bmatrix} 7\,a & 7\,b & 7\,c \\ 5g + 2\,d & 5h + 2\,e & 5i + 2\,f \\ d - 3\,a & e - 3\,b & f - 3\,c \end{bmatrix}$$

7 pts. *Be sure to show all steps in the computation.*

Answer:

$\det B =$

(2) We wish to solve the equation $AX = B$ where A, X, and B are given below. Express the value of y as a ratio of the determinants of two specific matrices. **Do not compute these determinants.** *6 pts.*

$$A = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 5 & 7 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 7 & 8 & 2 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Answer:

$$y = \frac{\det C}{\det D} \text{ where}$$

$C =$

$D =$

6 *pts.*

(3) Let A be as below and let $B = A^{-1}$. (It is given that A is invertible.) Express the $(3, 2)$ entry b_{32} of B as $(-1)^n$ times the ratio of the determinants of two specific matrices. **Do not compute these determinants.**

$$A = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 5 & 7 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 7 & 8 & 2 & 3 \end{bmatrix}$$

Answer:

$$b_{32} = (-1)^n \frac{\det C}{\det D} \text{ where}$$

$n =$

$C =$

$D =$

(4) Let

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 5 & 9 \\ 0 & 0 & 2 \end{bmatrix}$$

(a) Find the characteristic polynomial $p(\lambda)$ for A. *2 pts.*

(b) Find all eigenvalues for A. *2 pts.*

(c) Find a basis for each eigenspace of A. *6 pts.*

(5) Let

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

3 pts. (a) Verify that the vectors X, Y, and Z are eigenvectors for A where

$$X = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} \quad Y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad Z = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

2 pts. (b) What are the eigenvalues for A?

3 pts. (c) Find an eigenvector W for A corresponding to the eigenvalue 1 which has only *positive* entries. (*Note*: 0 is not positive!)

(6) For the matrix A in Problem 5, find a formula for $A^n B$ where *5 pts.*

$$B = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

(7) Suppose that A is a square matrix with characteristic polynomial $p(\lambda) = (\lambda - 1)(\lambda + 2)^4$.

2 pts. (a) Is A invertible? Why?

2 pts. (b) What are the possible dimensions for the $\lambda = -2$ eigenspace of A?

(8) Let

$$A = \begin{bmatrix} 3 & 4 & -2 \\ -3 & 8 & 0 \\ -3 & 1 & 7 \end{bmatrix}$$

It is given that X_1, X_2, and X_3 are eigenvectors for A corresponding respectively to the eigenvalues 5, 6, and 7 where

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad X_2 = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \quad X_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

Show that $\mathcal{B} = \{X_1, X_2, X_3\}$ is an eigenbasis for A, and find an **explicit** diagonal matrix D and an **explicit** invertible matrix Q such that $A = QDQ^{-1}$. (Hint: take D to be the diagonal matrix whose entries on the diagonal are the eigenvalues of A, Q the point matrix of the eigenbasis $\mathcal{B}$, and if calculating Q^{-1} via row reduction gets too complicated, use the formula for the inverse.)

$Q =$

$D =$

6 pts.

(9) Let A, B, and C be as below. Express $\det C$ in terms of $\det A$ and $\det B$. *This problem is meant to test your knowledge of the algebraic properties of determinants.* Do not compute any determinants to solve this problem. *Be sure to show all steps in your computation*

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & -1 & 3 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & -1 & 3 & 1 \\ 2 & 1 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix} C = \begin{bmatrix} 1 & -1 & 3 & 1 \\ 1 & 2 & 3 & 4 \\ 3 & 2 & 3 & 2 \\ 6 & 4 & 2 & 2 \end{bmatrix}$$

Answer:

$$\det C = x \det A + y \det B$$

where

$$x =$$

$$y =$$