MA 510 Solutions and Comments

3/7/10

(1) page 216 #9 (partial)

9a) using formula (6), p 212 for det A:
There are $n!$ permutations, each involves $(n-1)$ multiplications, so number is

$$n!(n-1)$$

b) Using (8) (cofactors)

$n=2$: $a_{11}a_{22} - a_{12}a_{21}$: two multiplications.

General n. Suppose Ω_n is the number using (8) (so $\Omega_2 = 2$). How do we go from Ω_{n-1} to Ω_n?

$n \times n$ $\begin{bmatrix} & a_{1j} & \\ & & \end{bmatrix}$ ← $(n-1)\times(n-1)$ cofactor

There are n terms to sum as, say, we go across 1st row. For a_{1j}, we multiply each of the products of Ω_{n-1} factors which occur in the with one more multiplication (we are multiplying by a_{1j} here), So the answer is

$$(*) \quad \underbrace{n}_{\#\text{ cols}} \cdot (\underbrace{\Omega_{n-1} + 1}_{\#\text{ multiplications in each term now}})$$

So if for $(n-1)$ we have $(n-1)!\left(1+\frac{1}{2!}+\cdots+\frac{1}{(n-2)!}\right)$ (and this <u>is</u> true when $n=2$), we would get from $(*)$:

$$n\left((n-1)!\left(1+\frac{1}{2!}+\cdots+\frac{1}{(n-2)!}\right)+1\right)$$

$$= n!\left(1+\frac{1}{2!}+\cdots+\frac{1}{(n-2)!}+\frac{1}{(n-1)!}\right) \quad \text{(check it out!)}$$

c) Reading page 14 is helpful here. If you do (and see me if not sure) we find that to get to Upper triangular form, we need $\frac{1}{3}(n^3-n)$ steps. (I think this was also done in lecture.) But once we have U, there are n pivots so we have $(n-1)$ multiplications. The total then is $\frac{1}{3}(n^3-n)+n-1 = \frac{1}{3}(n^3+2n-3)$ as required!