

(By request)

Prob 2 p344.

I look at

$$(*) \quad P(x) = x^T A x - 2x^T b = (x - A^{-1}b)^T A (x - A^{-1}b) + \text{const.}$$

I am not exactly sure what the author had in mind. But in high school we learn

$$x^2 - bx = (x-d)^2 + \text{const. if}$$

$$2d = b =$$

(so the constant is d^2). Let us do that here, using again d

$$x^T A x - 2x^T b = (x-d)^T A (x-d) + d^T A d$$

$$= x^T A x - 2x^T A d + d^T A d - d^T A d$$

So we need to agree with the formula (*) that

$$A d = b$$

$$\text{or } d = A^{-1}b$$

That explains why in (*) we have $(x - A^{-1}b)^T A (x - A^{-1}b)$.

Then

$$-d^T A d = -(A^{-1}b)^T A (A^{-1}b)$$

$$= -b^T A^{-1} A A^{-1} b$$

$$= -b^T A b$$

(recall $A^T = A, (A^{-1})^T = A^{-1}$)

So this means that

$$x^T A x - 2x^T b = (x - A^{-1}b)^T A (x - A^{-1}b) - b^T A b,$$

Thus: if we solve $Ax = b$, we have that

$$P(x) = -b^T A b = P_{\min}$$

(I call $2P$ what he calls P)

This seems to be what the author had in mind.