Name: ANSWERS **Math 511 January 21, 2010**

(*Work will be graded on the basis of clarity as well as accuracy.*)

(15) 1. True-False. Write T or F and give a reason in each case If the answer is F, it would be convincing to give an example. If an answer is 'True' and we proved it in class, you can refer to 'classwork.'

(a) A square matrix A may be factored F – need row exchanges sometimes. (The book hints at this – here is an example! $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $L = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix}$, $U = \begin{pmatrix} d & e \\ 0 & f \end{pmatrix}$)

$$A = LU$$

Then $LU = \begin{pmatrix} ad & ae \\ bd & be+cf \end{pmatrix}$ So if $A = LU$, $ad = 0$. If $a = 0$ then $ae = 0$, so

with $L(U)$ lower (upper) triangular. we could not have $a_{21} = 1$. If $d = 0$ then $bd = 0$ so we could not have $a_{21} = 1$. (The author should have made this clear!)

(b) Three linear equations in two unknowns can never have a solutions.

False

$$\begin{aligned} 3x + 2y &= 4 \\ 6x + 4y &= 8 \\ 9x + 12y &= 12 \end{aligned}$$

has infinitely many (there are other examples)

(c) If matrices A and B are both 3×3 and invertible, then BA is invertible.

True (classwork) $(BA)^{-1} = A^{-1}B^{-1}$

(15) 2. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 4 & 0 \end{pmatrix}.$$

(a) Write down the first two elementary matrices used to transform A to a matrix which is • triangular.

$E_{21}(2) = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad E_{31}(-2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

(b) These elementary matrices are • lower triangular.

(15) 3. Let A be a 2×3 matrix, B a 4×2 matrix and C a 3×5 matrix.

(a) Which products using each of these matrices once are permitted?

$B\ A\ C$

(b) What does the associative law say about this product (your answer should be an equation)? Is this equation true here? Give a reason if possible.

$(BA)C = B(AC)$

it is true (classwork)

(15) 4. Let A and B be 2×2 *lower* triangular matrices. Follow these steps to show that $C = AB$ is also lower triangluar.

(a) It is only necessary to show that $c_{ij} = 0$ [what are i and j here?].
Answer: $i = 1$; $j = 2$. $\begin{bmatrix} & * \\ & \end{bmatrix}$ this entry

(b)

$$c_{ij} = \sum_{k=1}^{2} a_{ik} b_{kj},$$

where i and j are from (a). This is a sum of • 2 terms. Explain why each term is zero, so that the full sum is zero [you can write this out without using the summation sign if you wish].

$a_{11} b_{12} + a_{12} b_{22}$

b_{12} = 0 since B lower triang.

a_{12} = 0 since A lower triang.