Name: SOLUTIONS **Math 511 Februrary 9, 2010**

(*Work will be graded on the basis of clarity as well as accuracy.*)

(12) 1. True-False. Write T or F and give a reason in each case If the answer is F, it would be convincing to give an example. If an answer is 'True' and we proved it in class, you can refer to 'classwork.'

(a) Let A be an $m \times n$ matrix. Then the solutions to the vector equation $Ax = 0$ form a vector space. True. If x_1, x_2 satisfy $Ax = 0$, so does $cx_1 + dx_2$.

(b) Same question as in (a) except we consider solutions to the vector equation

$$Ax = y, \qquad y = [1, 0, 0, 0, \dots, 0]^T,$$

where the first entry 1 is followed by $n - 1$ zeroes.

No If $Ax = y$, then $A(2x) = 2y \neq y$

(c) The solutions to the differential equation $f''(x) + f(x) = 0$ form a vector space.

Yes (classwork)

(d) The A be an $m \times n$ matrix whose rows are linearly independent. Then the zero vector is in the row space.

Yes! the zero vector is in any vector space/subspace

(8) 2. What is the dimension of the space of 2×3 matrices? Justify your answer by listing the elements of a basis. Is this the only basis possible?

Dimension is 6!

$$\begin{bmatrix}1&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\1&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&1\end{bmatrix}.$$

There are infinitely many bases, but they all have 6 elements.

(15) 3. Let A be the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

(a) Find the dimensions of the space $C(A)$ (column space), $N(A)$ (nullspace of A) and bases of each.

$C(A)$ has dimension 3, a basis is $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \\ 4 \end{pmatrix}$

$N(A)$: vectors y with $Ay = 0$. Row-reduced A is $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ (3 pivots)

So $N(A)$ has one dimension and y_4 is a free variable. So ~~we~~ start with the $\begin{pmatrix} \_ \\ \_ \\ 1 \\ \_ \end{pmatrix}$ free vari as basis and fill in the blanks, starting with the bottom equation. So we get for y

$$\begin{pmatrix} 4 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

(b) (**worth 5 points, don't use much time if you don't know this!**)Find two linearly independent vectors b so that the equation $Ay = b$ has no solution. Could b be the zero vector? A is already in reduced form. So linearly independent b's would be $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$