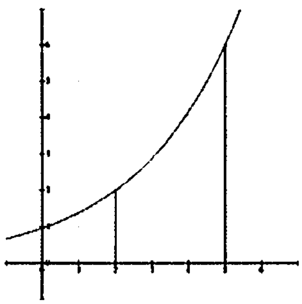
17 Friday, October 6

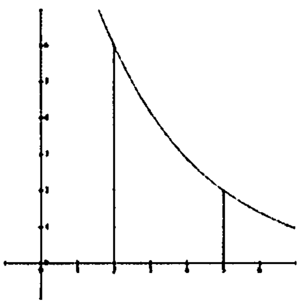
Increasing and Decreasing

Definition (Increasing and Decreasing). Let f be a function defined on (a, b), and let x_1, x_2 be any two numbers in (a, b).

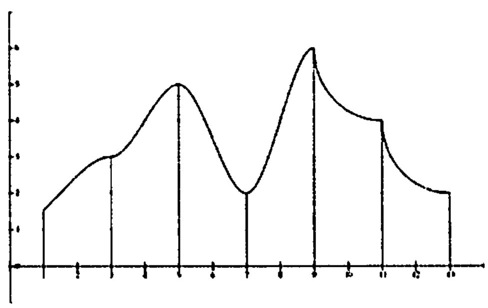
(i) f is **increasing** on (a, b) if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.

(ii) f is **decreasing** on (a, b) if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.





Example. Where is the function increasing and decreasing?



I: $(1, 5) \cup (7, 9)$

D: $(5, 7) \cup (9, 13)$

Theorem (First Derivative Test for Increasing and Decreasing). Suppose f is continuous on $[a,b]$ and differentiable on (a,b).

(i) If $f'(x) > 0$ for all x in (a,b), then f is increasing on $[a,b]$.

(ii) If $f'(x) < 0$ for all x in (a,b), then f is decreasing on $[a,b]$.

(iii) If $f'(x) = 0$ for all x in (a,b), then f is constant on $[a,b]$.

To find intervals of increasing and decreasing:

(1) Find all critical numbers c of f. Mark them on a number line.

(2) Pick test points from each interval and determine the sign of f' at those points.

(3) Apply the FDT to determine whether it is increasing or decreasing.

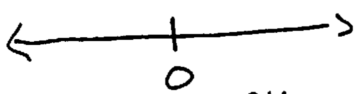
Example. Find the intervals where each function is increasing and decreasing.

(1) $f(x) = x^2$

- Find crit #'s.

$f'(x) = 2x = 0 \;\rightarrow\; x = 0$

- Plot on number line + test



$f'(-1) = -$ $\searrow$ $\qquad f'(1) = +$ $\nearrow$

I: $(0, \infty)$

D: $(-\infty, 0)$

(2) $f(x) = x^3 + 3x^2 - 9x + 10$

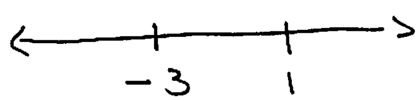
- Find crit #'s.

$f'(x) = 3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0 \;\rightarrow\; 3(x-1)(x+3) = 0$

$x = -3, 1$

- Plot on number line + test



$f'(-4) = +$ $\qquad f'(0) = -$ $\qquad f'(2) = +$

I: $(-\infty, -3) \cup (1, \infty)$

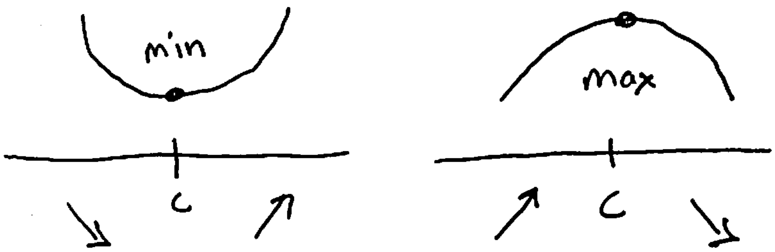
D: $(-3, 1)$.

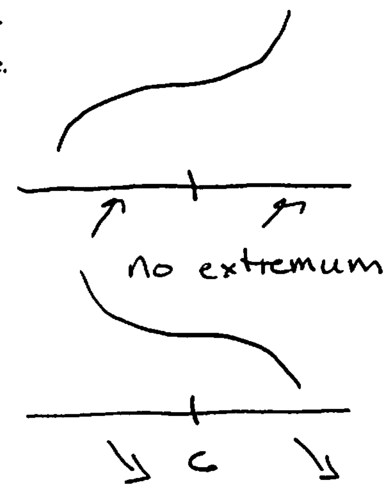
Theorem (First Derivative Test for Relative Extrema). Let c be a critical number of a function f that is continuous on (a, b) containing c. Then one of three cases occurs at c:

(i) If f' changes from negative to positive, then f has a relative minimum at c.

(ii) If f' changes from positive to negative, then f has a relative maximum at c.

(iii) If f' does not change signs, then f has no extremum at c.





To find relative extrema:

(1) Find all critical numbers c of f. Mark them on a number line.

(2) Determine increasing/decreasing.

(3) Apply FDT to identify the extremum.

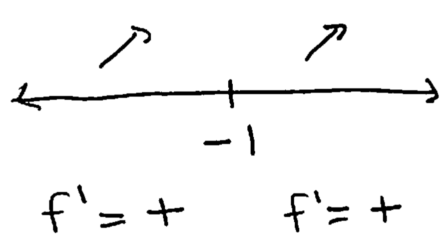
Example. Find the intervals where each function is increasing and decreasing. Identify the relative extrema.

(1) $f(x) = 2x^3 + 6x^2 + 6x - 4$

- Find crit. #'s.

$$f'(x) = 6x^2 + 12x + 6 = 0$$
$$6(x^2 + 2x + 1) = 0$$
$$6(x+1)^2 = 0.$$
$$x = -1$$

- Plot on number line



I: $(-\infty, \infty)$

D: NONE

$x = -1$ is not an extremum

(2) $f(x) = (x+1)^3(x-3)$

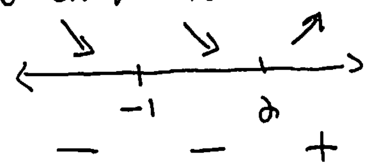
- Find crit #'s.

$$f'(x) = 3(x+1)^2(x-3) + (x+1)^3 = (x+1)^2[3(x-3)+(x+1)] = 0$$

$$(x+1)^2(4x-8) = 0$$

$$x = -1, 2$$

- Plot on number line



I: $(2, \infty)$

D: $(-\infty, 2)$

$x = 2$ is a min

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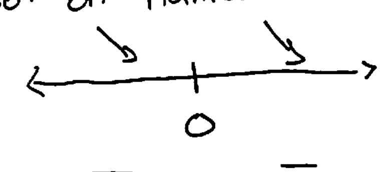
(3) $f(x) = \frac{1}{x}$

- Find crit #'s

$$f'(x) = -\frac{1}{x^2} = DNE \rightarrow \cancel{x=0} \text{ not in domain}$$

- Plot on number line



I: NONE

D: $(-\infty, 0) \cup (0, \infty)$

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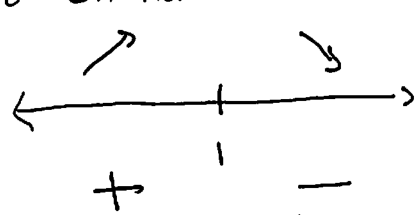
(4) $f(x) = \frac{1}{(x-1)^2} = (x-1)^{-2}$

- Find crit #'s.

$$f'(x) = \frac{-2}{(x-1)^3} = DNE \rightarrow \cancel{x=1} \text{ not in domain}$$

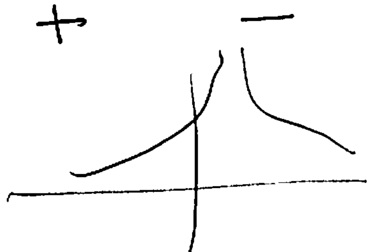
- Plot on number line



I: $(-\infty, 1)$

D: $(1, \infty)$

no extrema



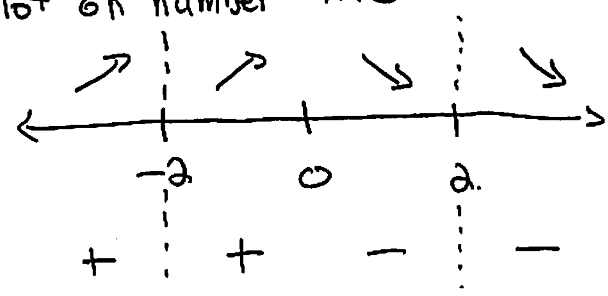
(5) $f(x) = \dfrac{1}{x-2} - \dfrac{1}{x+2}$

— Find crit #'s.

$$f'(x) = \frac{-1}{(x-2)^2} + \frac{1}{(x+2)^2} = \frac{-(x+2)^2 + (x-2)^2}{(x-2)^2(x+2)^2} = \frac{-8x}{(x-2)^2(x+2)^2}$$

$x = 0,\ \cancel{\pm 2}$ not in domain

— Plot on number line



I: $(-\infty, -2) \cup (-2, 0)$

D: $(0, 2) \cup (2, \infty)$

$x = 0$ is max

(6) $f(x) = \dfrac{1}{x^2+1}$

(7) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

(8) $f(x) = x^{2/3}(x+5) \quad = x^{5/3} + 5x^{2/3}$

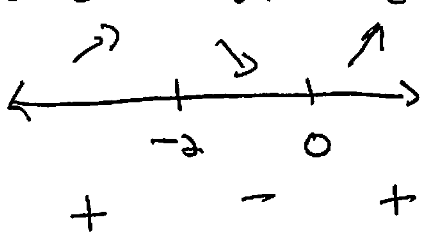
- Find crit #'s.

$$f'(x) = \frac{5}{3}x^{2/3} + \frac{10}{3}x^{-1/3} = x^{-1/3}\left(\frac{5}{3}x + \frac{10}{3}\right)$$

$$= \frac{5}{3}x^{-1/3}(x+2)$$

$$x = -2, 0$$

- Plot on number line



I: $(-\infty, -2) \cup (0, \infty)$

D: $(-2, 0)$

min $x = 0$

max $x = -2$.