2 Friday, August 25

Review

Definition (Informal Definition of Limit). Suppose $f(x)$ is defined in an open interval about $x = c$, *except possibly at $x = c$.* If the values of $f(x)$ become arbitrarily close to L as the values of x approach c from both sides, then the **limit** of $f(x)$ as x approaches c is L, or

$$\lim_{x \to c} f(x) = L.$$

Definition (One-sided Limit). If $f(x) \to L$ as $x \to c$ from x values that are to the left of c ($x < c$), then

$$\lim_{x \to c^-} f(x) = L.$$

Similarly, if $f(x) \to M$ as $x \to c$ from x values that are to the right of c ($x > c$), then

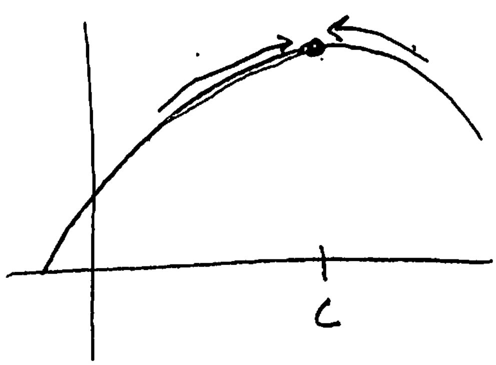
$$\lim_{x \to c^+} f(x) = M.$$

Theorem (Two-sided Limit Existence).

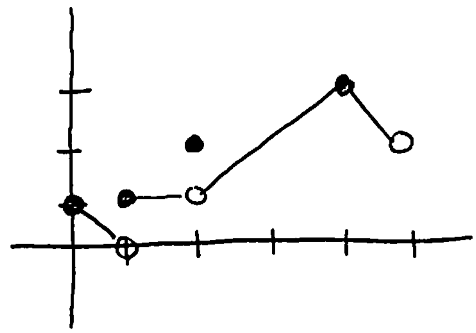
$$\lim_{x \to c} f(x) = L \quad \text{if and only if} \quad \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L.$$

Finding Limits Graphically

Given the graph of a function $f(x)$, the value of a one-sided limit at $x = c$ is the expected y-coordinate of the point if one was to trace the graph from the appropriate direction. The value of the two-sided limit is found by computing the one-sided limits and then applying the two-sided limit existence theorem. Remember, *the value of $f(x)$ at $x = c$ has no bearing on the existence or value of the limit.*



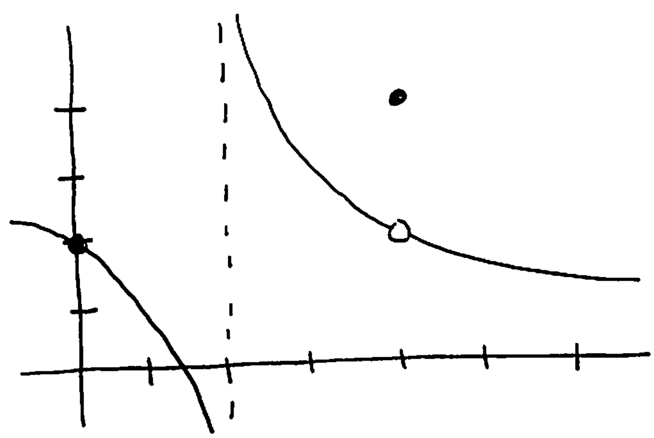
Ex:



c	$x \to c^-$	$x \to c^+$	$x \to c$	$f(c)$
0	DNE	1	DNE	1
1	0	1	DNE	1
2	1	1	1	2
3	2	2	2	2
4	3	3	3	3
5	2	DNE	DNE	DNE

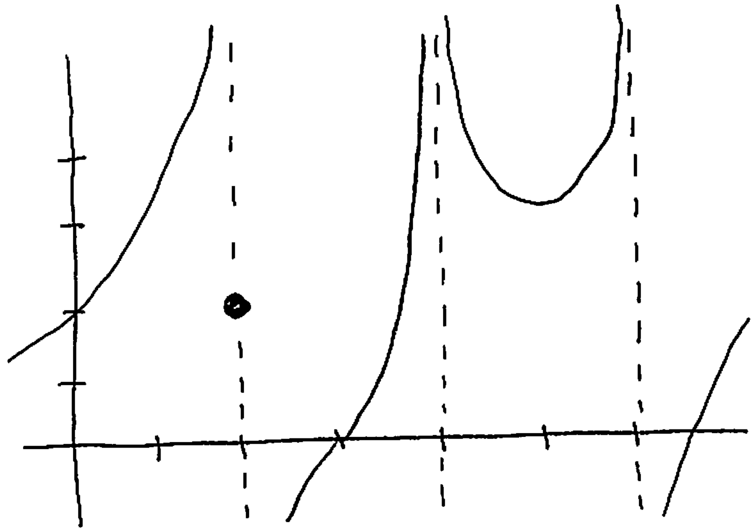
continuous

Ex:



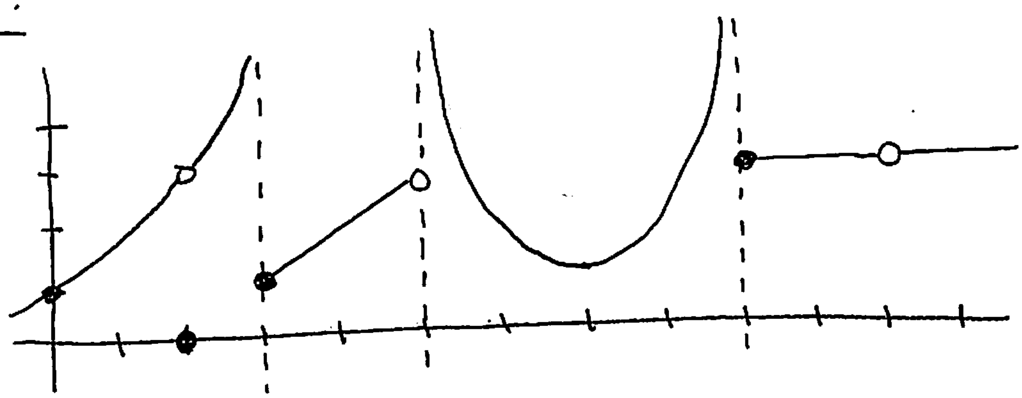
c	$f(c)$	$x \to c^-$	$x \to c^+$	$x \to c$
0	2	2	2	2
2	DNE	$-\infty$	$+\infty$	DNE
4	4	2	2	2

Ex:



c	$f(c)$	$x \to c^-$	$x \to c^+$	$x \to c$
2	2	$+\infty$	$-\infty$	DNE
4	DNE	$+\infty$	$+\infty$	$+\infty$
6	DNE	$+\infty$	$-\infty$	DNE

Ex:



c	$f(c)$	$x \to c^-$	$x \to c^+$	$x \to c$
0	1	1	1	1
2	0	3	3	3
3	1	$+\infty$	1	DNE
4	2	2	2	2
5	DNE	3	$+\infty$	DNE
7	1	1	1	1
9	3	$+\infty$	3	DNE
10	3	3	3	3
11	DNE	3	3	3