# 22 Monday, October 23

## Curve Sketching

Checklist for sketching curves:

(1) Find the domain of $f(x)$.

(2) Find $y$-intercept (where $x = 0$) and $x$-intercepts (where $f(x) = 0$).

(3) Find vertical and horizontal asymptotes. Find holes. Find slant asymptotes.

(4) Find $f'(x)$, critical numbers of $f$, and intervals of increasing and decreasing. Find horizontal tangents (where $f'(x) = 0$) and vertical tangents (where $f'(x) = \pm\infty$).

(5) Classify critical points as max/min via First Derivative Test.

(6) Find $f''(x)$, critical numbers of $f'$, intervals of concavity, and inflection points.

(7) Summarize on a number line. Plot key points and sketch the graph.

**Example 22.1.** Sketch the following functions.

(1) $f(x) = x^4 - 4x^3 = x^3(x-4)$

(1). Domain: $(-\infty, \infty)$.

(2). $y$-intercept: $(0,0)$.
$x$-intercept: $(0,0)$, $(4,0)$.

(3) No asymptotes or holes.

(4). Crit #'s of $f$ ($f'=0$ or DNE).

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$. $\rightarrow$ $x = 0, 3$.

Plot

sign $f'$ $-$ 0 $-$ 3 $+$

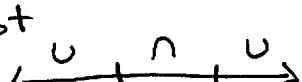
(5) $(0,0)$: no extrema, horizontal tangent

$(3,-27)$: min.

(6). Crit #'s of $f'$ ($f''=0$ or DNE).

$f''(x) = 12x^2 - 24x = 12x(x-2)$ $\rightarrow$ $x = 0, 2$.

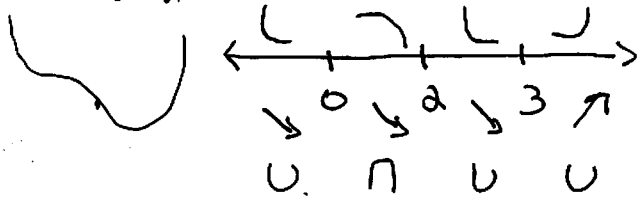
Plot

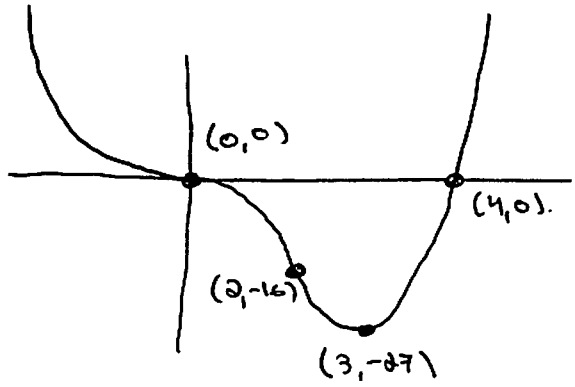


sign $f''$ $+$ 0 $-$ 2 $+$

IP:
$(0,0)$
$(2,-16)$

(7). Summarize





(2) $f(x) = \dfrac{2x^2+x-1}{x^2-1} = \dfrac{(2x-1)(x+1)}{(x-1)(x+1)}$

(1) Domain: $x \neq \pm 1$.

(2) y-intercept: $(0,1)$.

x-intercept: $(\frac{1}{2}, 0)$ $\quad 2x^2+x-1=0$, $(2x-1)(x+1)=0$

(3). HA: $y = \frac{2}{1} = 2$.

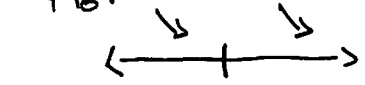
VA: $x = 1$.

Hole: $(-1, \frac{3}{2})$. $\quad \lim_{x\to -1} \dfrac{2x^2+x-1}{x^2-1} = \lim_{x\to -1} \dfrac{2x-1}{x-1} = \dfrac{3}{2}$

(4). Crit #'s of F ($f'=0$ or DNE).

$f'(x) = \dfrac{-1}{(x-1)^2}$ $\quad \to \quad x = \not{X}$ not in domain.

Plot



sign f' $\quad -$ | $-$

(5). no extrema.

(6). Crit #'s of f' ($f''=0$ or DNE).

$f''(x) = \dfrac{2}{(x-1)^3}$ $\quad \to \quad x = \not{X}$ not in domain.

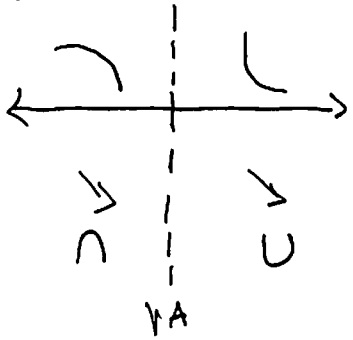
Plot

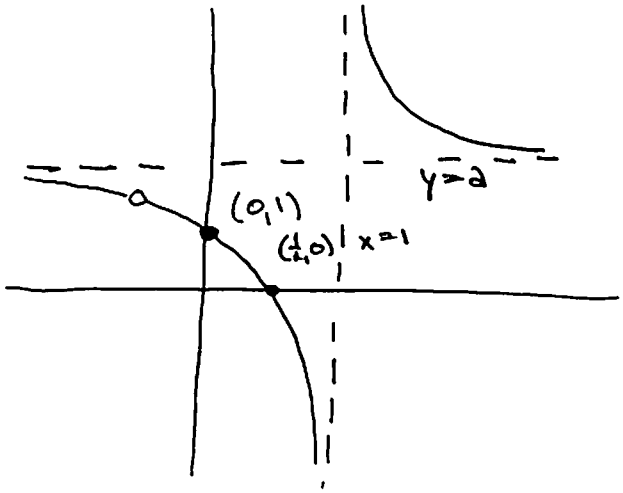
sign f'' $\quad -$ | $+$

IP:

none

(7) Summarize.





(3) $f(x) = x^{5/3} - 5x^{2/3} = x^{2/3}(x-5)$.

(1) Domain: $(-\infty, \infty)$.

(2) y-intercept: $(0,0)$

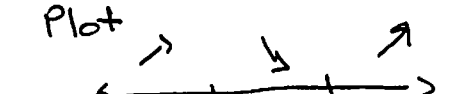
x-intercept: $(0,0)$, $(5,0)$.

(3) No asymptotes.

(4). Crit #'s of $f$ ($f'=0$ or DNE).

$f'(x) = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = \frac{5}{3}x^{-1/3}(x-2)$ $\rightarrow$ $x = 0, 2$.

Plot



sign $f'$ $+$ $0$ $-$ $2$ $+$

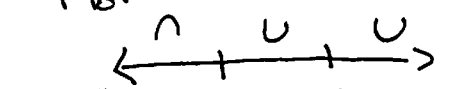
(5) $(0,0)$: max, vertical tangent

$(2, -4.8)$: min

(6). Crit #'s of $f'$ ($f''=0$ or DNE).

$f''(x) = \frac{10}{9}x^{-1/3} + \frac{10}{9}x^{-4/3} = \frac{10}{9}x^{-4/3}(x+1)$ $\rightarrow$ $x = 0, -1$.

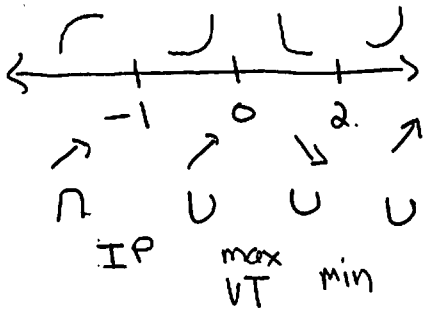
Plot

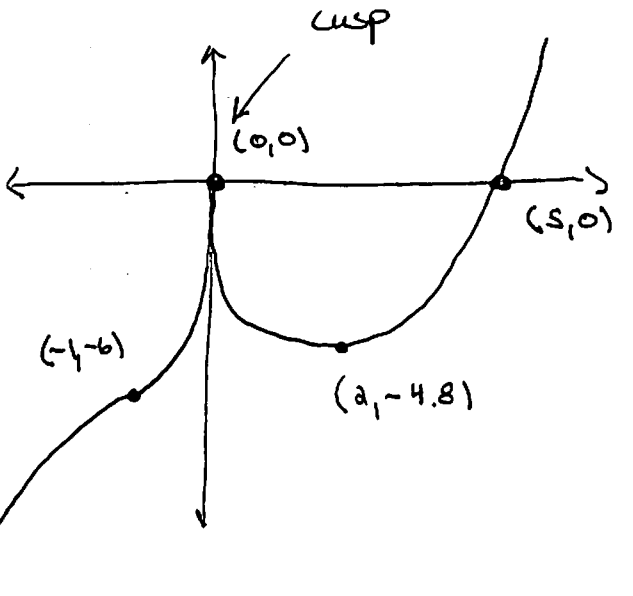


sign $f''$ $-$ $-1$ $+$ $0$ $+$

IP: $(-1, -6)$

(7). Summarize



(4) $f(x) = \dfrac{4x}{x^2 - 9}$

(5) $f(x) = \frac{x^3}{x^2-36} = x + \frac{36x}{x^2-36}$

$$\begin{array}{r|l} & x+0 \\ \hline x^2-36 & x^3+0x^2+0x+0 \\ (-) & x^3 \quad -36x \quad \downarrow \\ \hline & 36x+0 \end{array}$$

(1) Domain: $x \neq \pm 6$

(2) y-intercept: $(0,0)$

x-intercept: $(0,0)$.

(3) HA: none

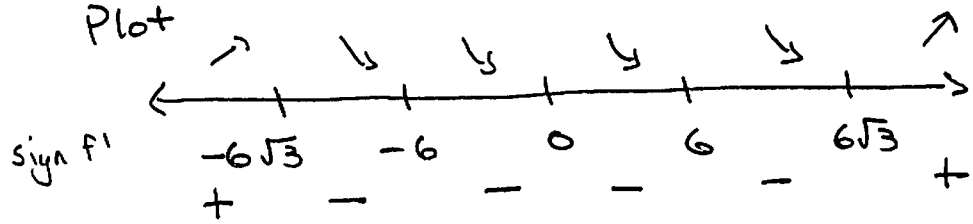
VA: $x = \pm 6$ No holes.

SA: $y = x$

(4). Crit #'s of $f$ ($f'=0$ or DNE).

$$f'(x) = \frac{x^4-108x^2}{(x^2-36)^2} \rightarrow x = \pm 6\sqrt{3}, 0, \cancel{\pm 6}$$ not in domain

Plot




(5). $(-6\sqrt{3}, -9\sqrt{3})$: max

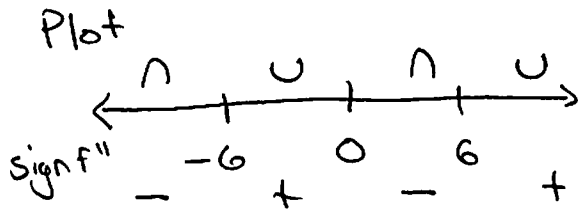
$(0,0)$ : neither

$(6\sqrt{3}, 9\sqrt{3})$: min.

(6). Crit #'s of $f'$ ($f''=0$ or DNE).

$$f''(x) = \frac{72x(x^2+108)}{(x^2-36)^3} \rightarrow x = 0, \cancel{\pm 6}$$ not in domain

Plot




IPs

$(0,0)$.

(6) $f(x) = x\sqrt{8 - x^2}$