Notes

9 Wednesday, September 13

Theorem (Quotient Rule).

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$$

Theorem (Derivatives of Trigonometric Functions).

(1) $\frac{d}{dx}[\tan x] = \sec^2 x$

(2) $\frac{d}{dx}[\sec x] = \sec x \tan x$

(3) $\frac{d}{dx}[\cot x] = -\csc^2 x$

(4) $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Example. Find the derivative.

(1) $y = \frac{2x+5}{2x-3}$ $\quad \frac{f}{g}$

$f' = 2$

$g' = 2$

$$y' = \frac{\overbrace{(2x-3)}^{g}\overbrace{(2)}^{f'} - \overbrace{(2x+5)}^{f}\overbrace{(2)}^{g'}}{(2x-3)^2} = \frac{-16}{(2x-3)^2}$$

(2) $y = \frac{t^2-1}{t^2+1}$ $\quad \frac{f}{g}$

$f' = 2t$

$g' = 2t$

$$y' = \frac{\overbrace{(t^2+1)}^{g}\overbrace{(2t)}^{f'} - \overbrace{(t^2-1)}^{f}\overbrace{(2t)}^{g'}}{(t^2+1)^2} = \frac{4t}{(t^2+1)^2}$$

(3) $z = \frac{2x+1}{x^2-1}$

$$z' = \frac{(x^2-1)(2) - (2x+1)(2x)}{(x^2-1)^2}$$

$$= \frac{-2x^2 - 2x - 2}{(x^2-1)^2}$$

(4) $s = \frac{t^2-1}{t^2+t-2}$

$$s' = \frac{(t^2+t-2)(2t) - (t^2-1)(2t+1)}{(t^2+t-2)^2}$$

$$= \cancel{2t^3} + 2t^2 - 4t - \cancel{2t^3} + 2t - t^2 + 1$$

$$= \frac{t^2 - 2t + 1}{(t^2+t-2)^2} = \frac{(t-1)^2}{(t-1)^2(t+2)^2} = \frac{1}{(t+2)^2}$$

$$s = \frac{\cancel{(t-1)}(t+1)}{\cancel{(t-1)}(t+2)} = \frac{t+1}{t+2}$$

$$s' = \frac{(t+2) - (t+1)}{(t+2)^2} = \frac{1}{(t+2)^2}$$

(5) $r = \frac{\sqrt{s}-1}{\sqrt{s}+1}$

$$r' = \frac{(s^{1/2}+1)\left(\frac{1}{2}s^{-1/2}\right) - (s^{1/2}-1)\left(\frac{1}{2}s^{-1/2}\right)}{(s^{1/2}+1)^2}$$

$$= \frac{s^{-1/2}}{(s^{1/2}+1)^2}$$

(6) $u = \frac{5x+1}{2\sqrt{x}}$

$$u' = \frac{2x^{1/2}(5) - (5x+1)x^{-1/2}}{4x} = \frac{10x^{1/2} - 5x^{1/2} - x^{-1/2}}{4x}$$

$$= \frac{5}{4}x^{-1/2} - \frac{1}{4}x^{-3/2}$$

$$u = \frac{5}{2}x^{1/2} + \frac{1}{2}x^{-1/2}$$

$$u' = \frac{5}{4}x^{-1/2} - \frac{1}{4}x^{-3/2}$$

(7) $y = \frac{\cos x}{x} + \frac{x}{\cos x}$

$$y' = \frac{x(-\sin x) - \cos x}{x^2} + \frac{\cos x - x(-\sin x)}{\cos^2 x}$$

$$= \frac{-x\sin x - \cos x}{x^2} + \frac{\cos x + x\sin x}{\cos^2 x}$$

(8) $y = \frac{1}{(x^2+x+1)e^x}$

$$y' = \frac{\cancel{(x^2+x+1)e^x \cdot (0)} - \left[(x^2+x+1)e^x + (2x+1)e^x\right]}{(x^2+x+1)^2 e^{2x}}$$

$$= \frac{-(x^2+3x+2)e^x}{(x^2+x+1)^2 e^{2x}}$$

$$= \frac{-(x^2+3x+2)}{(x^2+x+1)^2 e^x}$$

(9) $s = \frac{1+\csc t}{1-\csc t}$

$$s' = \frac{(1-\csc t)(-\csc t \cot t) - (1+\csc t)(\csc t \cot t)}{(1-\csc t)^2}$$

$$= \frac{-2\csc t \cot t}{(1-\csc t)^2}$$

(10) $s = \frac{\sin t}{1-\cos t}$

$$s' = \frac{(1-\cos t)\cos t - (\sin t)(\sin t)}{(1-\cos t)^2}$$

$$= \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2}$$

$$= \frac{\cos t - 1}{(1-\cos t)^2}$$

$$= \frac{-1}{1-\cos t}$$

$$= \frac{1}{\cos t - 1}$$

(11) $y = \frac{\sin x + \cos x}{\cos x}$

$$y' = \frac{\cos x\,(\cos x - \sin x) - (\sin x + \cos x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x - \cancel{\cos x}\sin x + \sin^2 x + \cancel{\cos x}\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$$y = \frac{\sin x + \cos x}{\cos x} = \tan x + 1$$

$$y' = \sec^2 x$$

(12) $y = \frac{x^3 \cos x}{e^x}$

$$y' = \frac{e^x\left[3x^2\cos x - x^3 \sin x\right] - x^3 e^x \cos x}{e^{2x}}$$

$$= \frac{3x^2 \cos x - x^3 \sin x - x^3 \cos x}{e^x}$$