11 Monday, September 18

Chain Rule

Theorem (Chain Rule)**.** Let $y = f(u)$ be differentiable with respect to u, and let $u = g(x)$ be differentiable with respect to x. Then $y = f(g(x))$ is differentiable with respect to x and

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'\left(g(x)\right) \cdot g'(x) \qquad \text{or} \qquad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

Example (Using Chain Rule with Transcendental Functions)**.** Find $f'(x)$.

(1) $f(x) = (\sin x - 1)^2$

(2) $f(x) = \sqrt[3]{1 + \cos(2x)}$

(3) $f(x) = \sec(4x - 1)$

(4) $f(x) = \tan\left[(2t+5)^{-2/3}\right]$

(5) $f(x) = \cos\left(e^{-x^2}\right)$

(6) $f(x) = e^{\sin t}\,(2x+1)$

(7) $f(x) = \tan\left(e^{4x}\right)$

(8) $f(x) = e^{-x}\sin(3x)$

(9) $f(x) = \left(8 + \csc^2 x\right)^4$

Theorem 11.1 (Derivative of $\ln x$)**.** *If u is a differentiable function of x, then*

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \qquad \frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}.$$

Example. Find $f'(x)$.

(1) $f(x) = \ln 3x$

(2) $f(x) = \ln kx$, k a constant

(3) $f(x) = (\ln x)^3$

(4) $f(x) = \frac{\ln x}{x}$

(5) $f(x) = \frac{\ln x}{1 + \ln x}$

(6) $f(x) = \ln(\ln x)$

(7) $f(x) = \ln(\sec x + \tan x)$

(8) $f(x) = \ln\left(3xe^{-x}\right)$

(9) $f(x) = e^{\cos x + \ln x}$

(10) $f(x) = \ln\left(\frac{e^x}{1+e^x}\right)$

(11) $f(x) = \ln(\sec(\ln x))$

(12) $f(x) = \frac{1}{2}\ln\left(\cos^2(8x)\right)$