12 Wednesday, September 20

Higher Order Derivatives

Definition (Higher Order Derivatives)**.** The derivative $y' = dy/dx$ is the **first (order) derivative** of y with respect to x. The derivative itself is a function of x, and can be differentiated again to produce

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d^2y}{dx^2}.$$

This is the **second (order) derivative** of y. This process can be continued with the third derivative, fourth derivative, and so on. The arbitrary n**th (order) derivative** is denoted

$$y^{(n)} = \frac{d^n y}{dx^n}.$$

Example 12.1. Find the second and third derivatives.

(1) $y = x^2 + x + 8$

(2) $w = 3z^7 - 7z^3 + 21z^2$

(3) $y = \dfrac{1}{\sqrt{x}}$

(4) $y = e^x$

(5) $y = \sin x$

(6) $y = \dfrac{\ln x}{e^x}$

Example 12.2. Find $y^{(101)}$.

(1) $y = x^3 - 2x + 1$

(2) $y = x^{102} - x^{101} + x^{99} - 7$

(3) $y = 3e^x$

(4) $y = 2\cos x$

(5) $y = \sin(3x)$

Definition (Acceleration)**. Acceleration** is the derivative of velocity with respect to time. If the position is given by $s = f(t)$, then the acceleration is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

Example 12.3.

(1) A rock is thrown vertically upward, reaching a height of $s = 24t - 4.9t^2$ meters after t seconds.

(a) Find the rock's velocity and acceleration at time t.

(b) How long would it take the rock to reach its highest point?

(c) How high would the rock go?

(2) The position of an object is given by the function $f(t) = 3e^{-t} \sin t$.

(a) Find the object's velocity and acceleration at time t.

(b) What is the object's velocity and acceleration at time $t = \pi$? $t = \pi/4$?

(c) When is the object at a standstill?