14 Friday, September 29 and Monday, October 2

Related Rates

A related rate problem is an analysis of how the rate of change of one variable affects the rate of change of other variables in some situation. Solving such problems can be summarized with the following steps:

(1) Draw a picture and name the variables and constants. Use t for time, and assume all variables are functions of t.

(2) Write down the numerical information in terms of the symbols you chose.

(3) Write down what you are asked to find.

(4) Write an equation(s) that relates the variables.

(5) Differentiate (implicitly) with respect to t.

(6) Evaluate with known information.

Example.

(1) Assume that x and y are both differentiable functions of t and find the required values of dy/dt and dx/dt if x and y are related by

$$x^3y + y^3x = -30.$$

(a) Find dx/dt if $x = 3$, $y = -1$, and $dy/dt = 1$.

(b) Find dy/dt if $x = -1$, $y = 3$, and $dx/dt = 1/2$.

(2) When a circular plate of metal is heated in an oven, its radius increases at the rate of $0.01\,\text{cm/min}$. At what rate is the plate's area increasing when the radius is $50\,\text{cm}$?

(3) The length x of a rectangle is decreasing at the rate of $2\,\text{cm/sec}$ while the width y is incrasing at the rate of $2\,\text{cm/sec}$. When $x = 12\,\text{cm}$ and $y = 5\,\text{cm}$, Find the rate of change of

(a) the area,

(b) the perimeter,

(c) and the length of the diagonal.

(4) A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

(5) How rapidly will the fluid level inside a vertical cylindrical tank of radius $1\,\mathrm{m}$ drop if we pump the fluid out at the rate of $3000\,\mathrm{L/min}$?

(6) The voltage V, current I, and resistance R of a circuit is related by $V = IR$. Suppose that V is increasing at a rate of $1\,\mathrm{V/sec}$ while I is decreasing at $1/3\,\mathrm{A/sec}$. Find the rate at which R is changing when $V = 12\,\mathrm{V}$ and $I = 2\,\mathrm{A}$.

(7) A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?

(8) A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

(9) Alice and Bob are walking on straight streets that meet at right angles. Alice approaches the intersection at 2 m/sec; Bob moves away from the intersection 1 m/sec. At what rate is the angle θ changing when Alice is 10 m from the intersection and Bob is 20 m from the intersection?

(10) A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a cyclist moving at a constant rate of 17 ft/sec pass under it. How fast is the distance between the cyclist and the balloon increasing 3 sec later?

(11) Coffee is draining from a conical filter of radius 3" and height 6" into a cylindrical coffeepot at the rate of $10\,\text{in}^3/\text{min}$. How fast is the level in the pot rising and filter falling when the coffee level is 5"?

(12) You are videotaping a race from a stand 132 ft from the track, following a car that is moving at 264 ft/sec. How fast will your camera angle θ be changing when the car is right in front of you?

(13) A baseball diamond is a square 90 ft on a side. A player runs from first base to second at a rate of 16 ft/sec.

(a) At what rate is the player's distance from third base changing when the play is 30 ft from first base?

(b) At what rates are the angles θ_1 and θ_2 changing at that time?