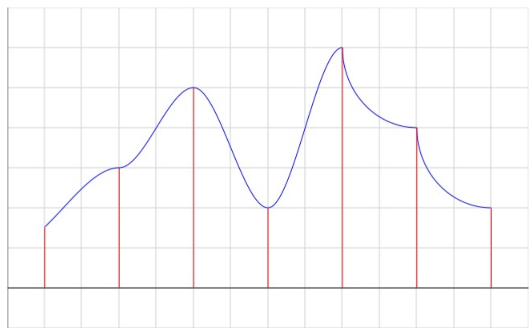
16 Wednesday, October 4

Relative Extrema and Critical Numbers

Definition (Relative Extrema)**.** f has a **relative maximum** at $x = c$ if $f(c) \geq f(x)$ for all x in some interval (a, b) containing c. f has a **relative minimum** at $x = c$ if $f(c) \leq f(x)$ for all x in some interval (a, b) containing c. Relative extrema are also called **local extrema**.

Observation. Consider the following graph.



Note that points where there are extrema, the derivative of f is either 0 or does not exist.

Theorem 16.1 (First Derivative Test for Relative Extrema)**.** *If f has a local maximum or local minimum at an interior point $x = c$, then either $f'(c) = 0$ or does not exist.*

Definition (Critical Numbers). If $f'(c) = 0$ or f is not differentiable at an interior point $x = c$, then c is a **critical number**.

Example. Find the critical numbers of each function in their domains.

(1) $f(x) = x^2 - 1$

(2) $f(x) = x - 5$

(3) $f(x) = 8x^2 - x^4$

(4) $f(x) = \sqrt[3]{x}$

(5) $f(x) = 2\cos x + \sin(2x)$

(6) $f(x) = \cot x + 2\csc x$

(7) $f(x) = x\sqrt{4 - x^2}$

(8) $f(x) = \dfrac{x}{x^2 - x + 1}$

(9) $f(x) = \ln(x^2 + x + 1)$

(10) $f(x) = x^{3/4} - 2x^{1/4}$

(11) $f(x) = x^{4/5}(x - 4)^2$

(12) $f(x) = |x|$

(13) $f(x) = |3x - 4|$

(14) $f(x) = xe^{-x^2/8}$

(15) $f(x) = x - 2\tan^{-1} x$