# 19 Monday, October 16

## Absolute Extrema on an Interval

**Definition** (Absolute Extrema)**.** Let $f$ be defined on an interval $I$ containing $c$.

(i) $f(c)$ is the **minimum of** $f$ **on** $I$ when $f(c) \leq f(x)$ for all $x$ in $I$.

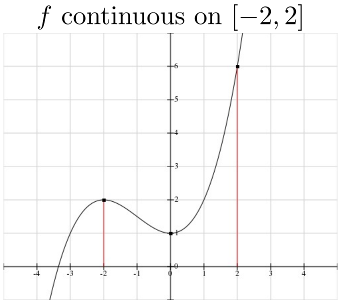
(ii) $f(c)$ is the **maximum of** $f$ **on** $I$ when $f(c) \geq f(x)$ for all $x$ in $I$.

The maximum and minimum values are the **extreme values**, or **extrema**. In particular, the maximum and minimum of $f$ on $I$ are the **absolute maximum** and **absolute minimum** of $f$ on $I$ (**global** is also used for absolute). If extrema occur at endpoints, then they are called an **endpoint extrema**.

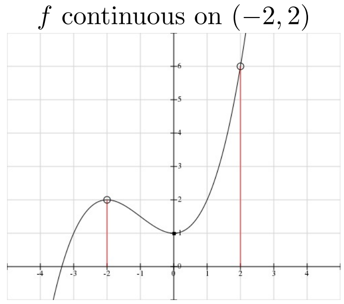
**Theorem** (Extreme Value Theorem)**.** If $f$ is continuous on a closed interval $[a, b]$, then $f$ attains both a maximum and minimum on the interval.

**Note.** The continuity condition on $[a, b]$ is necessary for the EVT to hold.

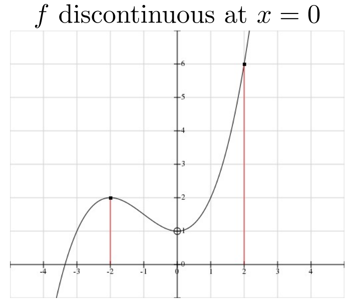
$f$ continuous on $[-2, 2]$



$f$ continuous on $(-2, 2)$



$f$ discontinuous at $x = 0$



To find absolute extrema of a continuous function $f$ on $[a, b]$:

(1) Find the critical numbers of $f$ that lie in $[a, b]$

(2) Evaluate $f$ at each critical number.

(3) Evaluate $f$ at the endpoints $a$ and $b$.

(4) The max/min of these numbers is the absolute extrema.

**Example.** Find the absolute extrema of each function on the given interval.

(1) $f(x) = 4 - x^2$ on $[-3, 1]$

(2) $f(x) = x^3 - \frac{3}{2}x^2$ on $[-1, 2]$

(3) $f(x) = -\dfrac{1}{x^2}$ on $[1, 2]$

(4) $f(x) = x + \dfrac{1}{x}$ on $\left[\dfrac{1}{2}, 3\right]$

(5) $f(x) = \dfrac{3}{x-5}$ on $[0, 2]$

(6) $f(x) = \dfrac{x^2}{x+1}$ on $\left[-\dfrac{1}{2}, 1\right]$

(7) $f(x) = \sqrt{4 - x^2}$ on $[-1, 2]$

(8) $f(x) = 3x^{2/3} - 2x$ on $[-1, 1]$

(9) $f(x) = \sec x$ on $\left[-\frac{\pi}{3}, \frac{\pi}{6}\right]$

(10) $f(x) = x^2 e^x$ on $[-3, 1]$

(11) $f(x) = 5e^x \sin x$ on $[0, \pi]$

(12) $f(x) = x^2 - 8 \ln x$ on $[1, 6]$

(13) $f(x) = x \ln(x+3)$ on $[0, 3]$

(14) $f(x) = \dfrac{\ln x}{8x}$ on $[1, 4]$