22 Monday, October 23

Curve Sketching

Checklist for sketching curves:

- (1) Find the domain of f(x).
- (2) Find y-intercept (where x = 0) and x-intercepts (where f(x) = 0).
- (3) Find vertical and horizontal asymptotes. Find holes.
- (4) Find f'(x), critical numbers of f, and intervals of increasing and decreasing. Find horizontal tangents (where f'(x) = 0) and vertical tangents (where $f'(x) = \pm \infty$).
- (5) Classify critical points as max/min via First Derivative Test.
- (6) Find f''(x), critical numbers of f', intervals of concavity, and inflection points.
- (7) Summarize on a number line. Plot key points and sketch the graph.

Example 22.1. Sketch the following functions.

(1) $f(x) = x^4 - 4x^3$

(2)
$$f(x) = \frac{2x^2 + x - 1}{x^2 - 1}$$

(3) $f(x) = x^{5/3} - 5x^{2/3}$

(4)
$$f(x) = \frac{4x}{x^2 - 9}$$

(5)
$$f(x) = \frac{x^3}{x^2 - 36}$$

(6) $f(x) = x\sqrt{8 - x^2}$

(7)
$$f(x) = \frac{x^2 - 49}{x^2 + 5x - 14}$$

(8)
$$f(x) = \frac{x-1}{x^3 - 2x^2}$$

(9)
$$f(x) = \frac{3}{4}(x^2 - 1)^{2/3}$$