28 Monday, November 6

Sigma Notation

Definition (Sigma Notation for Finite Sums). The symbol Σ is used as shorthand to denote a sum of numbers following a pattern:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \dots + a_n$$

The a_k are the **terms** of the sum: a_1 is the first term, a_2 is the second term, a_k is the *k***th term**, and a_n is the last term. The variable k is the **index of summation**. When computing the sum, k runs through the values from 1 (determined by the k = 1 below the Σ) to n (determined by n above the Σ). The 1 is the **lower limit of summation** and the n is the **upper limit of summation**.

Example. Evaluate the following sums.

(1)
$$\sum_{k=1}^{5} k^2$$

(2)
$$\sum_{k=-1}^{3} (k^3 - 1)$$

$$(3) \sum_{k=-1}^{1} e^k$$

$$(4) \ \sum_{k=2}^{5} \frac{(-1)^k}{k}$$

(5)
$$\sum_{k=0}^{3} \left(3 - \left(\frac{2}{3}\right)^{k}\right)$$

Riemann Sums

Definition (Riemann Sum). A **Riemann Sum** is a particular summing operation done on a function over a closed interval. The simplest example of a Riemann sum is given by an attempt to approximate the area between the graph of a function and the *x*-axis.

Example. Use the left and right Riemann sums to approximate the area under the following curves. (1) y = x over [1,3] with 4 rectangles (2) y = 3x - 4 over [-3, 5] with 32 rectangles

(3) $y = x^2 + 1$ over [-1, 1] with 4 rectangles

(4) $y = x^3 + x$ over [0, 4] with 8 rectangles

(5) $y = \cos x$ over $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with 4 rectangles

(6)
$$y = \frac{\sin x}{x}$$
 over [1, 11] with 50 rectangles

(7) $y = \sqrt{1 + x^4}$ over [0, 50] with 200 rectangles