32 Friday, November 17

The Fundamental Theorem of Calculus

Example. Find the area of the regions enclosed by the following curves.

(1) $y = x^2 - 3x - 4$, y = 0

(2)
$$y = \frac{1}{x} - \frac{1}{x^2}, \quad y = 0, \quad x = 2$$

(3)
$$y = 1$$
, $y = \sqrt[4]{x}$, $x = 0$

Theorem (Net Change Theorem). The integral of a rate of change is the net change:

$$\int_{a}^{b} F'(x) \, dx = F(b) - F(a)$$

Example.

(1) The rate of change of the population of a city is given by

$$\frac{dP}{dt} = t - 4, \quad 0 \le t \le 8$$

in millions per year. By how much did the population increase over 4 years?

(2) Water flows out of a tank at a rate of $100 - t^2$ liters per minute. How much water flowed out of the tank in the first 6 minutes?

(3) The velocity of a particle is given in feet per second by

$$v(t) = 2\sqrt{t}, 4 \le t \le 9$$

- (a) What is the displacement over the interval?
- (b) What is the total distance the particle travels over the interval?

(4) The velocity of a particle is given in feet per second by

 $v(t) = t^2 - 16, 0 \le t \le 9$

- (a) What is the displacement over the interval?
- (b) What is the total distance the particle travels over the interval?

(5) The velocity of a particle is given in feet per second by

$$v(t) = (t-2)^2(t-4)$$

- (a) What is the displacement over the interval?
- (b) What is the total distance the particle travels over the interval?

(6) The velocity of a particle is given in feet per second by

 $v(t) = 2\sin t, 0 \le t \le 3\pi/2$

- (a) What is the displacement over the interval?
- (b) What is the total distance the particle travels over the interval?

- (7) An oil storage tank ruptures at a time t = 0 and oil leaks from the tank at a rate of r(t) = 3/t liters per minute.
 - (a) How much oil leaks out during the first hour?
 - (b) How much oil leaks out during the third hour?