34 Monday, December 4

Review: Law of Exponential Growth/Decay

Theorem (Law of Exponential Growth/Decay)**.** If y is a differentiable function of t such that $y > 0$ and $y' = ky$ for some constant k, then

$$y = Ce^{kt}$$

where C is the **initial value** of y (the value of y at $t = 0$) and k is the **proportionality constant**. If $k > 0$, then y is growing exponentially. If $k < 0$, then y is decaying exponentially.

Example. Solve the differential equations.

(1) $\dfrac{dy}{dt} = -10y, y(0) = 20$

(2) $\dfrac{dy}{dt} = -3y, y(0) = -5$

Definition. In radioactive decay, the **half-life** of a radioactive element is the time required for half of the radioactive element to decay. In particular,

$$\text{half-life} = \frac{\ln 2}{k}$$

Example.

(1) The decay equation for radon-222 gas is known to be $y = y_0 e^{-0.18t}$. What is the half-life of radon-222? About how long will it take the radon in a sealed sample of air to fall to 90% of its original value?

(2) The half-life of polonium is 139 days, but your sample will not be useful to you after 95% of the radioactive nuclei present on the day the sample arrives has disintegrated. For about how many days after the sample arrives will you be able to use the polonium?

(3) The half-life of plutonium is $24,100$ years. If after $20,000$ years there is $1.2\,\mathrm{g}$ of a sample, then how much was there originally?

(4) The half-life of carbon-14 is about 5700 years. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

(5) The half-life of radon-226 is about 1600 years. If there was originally 20 g, how much will there be 5000 years?

(6) The rate of processing of raw sugar is proportional to the amount of raw sugar remaining. If 1000 kg of raw sugar reduces to 800 kg of raw sugar during the first 10 h, how much raw sugar will remain after another 14 h?