6 Wednesday, September 6

Theorem (Derivative Rules)**.** Let $f(x), g(x)$ be differentiable and k some constant.

(1) Constant Rule: $\dfrac{d}{dx}[c] = 0$

(2) Power Rule: $\dfrac{d}{dx}[x^n] = nx^{n-1}$ where n is any real number

(3) Constant Multiple Rule: $\dfrac{d}{dx}\left[kf(x)\right] = kf'(x)$

(4) Sum/Difference Rule: $\dfrac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$

(5) Sine/Cosine Rule: $\frac{d}{dx}\left[\sin x\right] = \cos x$ $\qquad$ $\frac{d}{dx}\left[\cos x\right] = -\sin x$

(6) Exponential Rule: $\frac{d}{dx}\left[e^x\right] = e^x$

Example. Find the derivative of the following.

(1) $y = -x^2 + 3$

(2) $s = 5t^3 - 3t^5$

(3) $w = 3z^7 - 7z^3 + 21z^2$

(4) $y = \dfrac{4x^3}{3} - x$

(5) $y = \dfrac{x^3}{3} + \dfrac{x^2}{2} + \dfrac{x}{4}$

(6) $w = 3z^{-2} - z^{-1} - 3$

(7) $y = 6x^2 - 10x - 5x^{-2}$

(8) $y = 4 - 2x - x^{-3}$

(9) $r = \frac{1}{3s^2} - \frac{5}{2s}$

(10) $r = \frac{1}{\sqrt{\theta}} + \sqrt{\theta}$

(11) $y = \sqrt{x^3} + \frac{1}{x^2} - \sqrt{2}$

(12) $w = \frac{z^3 - 5z + 1}{z}$

(13) $y = -10x + 3\cos x$

(14) $y = \frac{3}{x} + 5\sin x$

(15) $y = x^3 - e^x + \sin x$

Example. Find the equation for the tangent line to the graph of the given function at the given point.

(1) $f(x) = x^2 + x + 8$ at $x = -1$

(2) $g(t) = -2t^{-1} + \frac{4}{t^2} + 1$ at $t = 1$

(3) $r(\theta) = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$ at $\theta = -2$

(4) $h(x) = \frac{3}{x} + \frac{2}{\sqrt{x}} - \frac{x}{3}$ at $x = 4$

(5) $k(x) = \cos x - \sin x$ at $x = \pi$

(6) $f(x) = 4x - 3\sin x + 2e^x$ at $x = 0$

Example 6.1. Where do the following functions have a horizontal tangent?

(1) $f(x) = x^5 - 5x$

(2) $f(x) = x + \dfrac{1}{x}$

(3) $f(x) = (x+1)^3 x$

(4) $f(x) = x^{2/3}(x+5)$

(5) $f(x) = x - \sin x$

(6) $f(x) = 4e^x - 1$