

7 Friday, September 8

Review (Rates of Change). The **average rate of change** of a function $f(x)$ with respect to x over the interval x_0 to $x_0 + \Delta x$ is

$$\text{Rate}_{\text{avg}} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

The **(instantaneous) rate of change** of f with respect to x at x_0 is the derivative $f'(x_0)$.

Example.

- (1) How fast is the area of a circle changing with respect to the radius when the radius is $r = 5$? What is the average rate of change from $r = 5$ to $r = 5.5$?
- (2) The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$. How fast is the water running out the end at 10 min? What is the average rate at which the water flows out during the first 10 min?
- (3) Suppose the cost of producing x washing machines is $c(x) = 2000 + 100x - 0.1x^2$. What is the average cost per machine of producing the first 100 machines? What is the rate of change of the cost when 100 machines are produced?

Definition (Displacement and Velocity). Suppose that an object is moving along a line such that its position at time t is given by $s = f(t)$. The **displacement** of the object over the time interval t to $t + \Delta t$ is

$$\Delta s = f(t + \Delta t) - f(t),$$

and the **average velocity** of the object is the average rate of change of the position, or

$$v_{\text{avg}} = \frac{\text{displacement}}{\text{time interval}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

The **(instantaneous) velocity** is the derivative of the position $s = f(t)$,

$$v(t) = \frac{ds}{dt}.$$

The **speed** is the absolute value of velocity, $|v(t)|$.

Example.

(1) A rock is blasted straight up in the air, and its height above the ground is given by $s = 160t - 16t^2$ feet.

(a) How high does the rock go?

(b) What is the velocity and speed of the rock at $t = 1$ and $t = 3$ sec? What is the rock's average velocity in this time interval?

(c) What is the velocity and speed of the rock when it is 256 feet above the ground on the way up? on the way down?

(d) When does the rock hit the ground again?

(2) An astronaut throws a ball up in the air from the surface of the moon. Its height is given by $s = 24t - 0.8t^2$ meters in t seconds.

(a) Find the ball's velocity at time t .

(b) How long does it take the ball to reach its highest point?

(c) How high does the ball go?

(d) How long does it take the ball to reach half its maximum height?

(e) How long is the ball aloft?

- (3) A body moving on a coordinate line has position given by $s = 2 - 2 \sin t$. What is the body's velocity and speed at time $t = \pi/4$? What is its average velocity from $t = 0$ to $t = \pi/4$.

- (4) Another body has position $s = \sin t + \cos t$. What is the body's velocity and speed at time $t = \pi/3$? What is its average velocity from $t = 0$ to $t = \pi/3$?