Solutions

1 Ans a) slope of tangent line is $f'(x) = 6x^2+6x-12$.

$f'(x)=0 \Leftrightarrow$ tangent line at $(x, f(x))$ is horizontal.

and $f'(x) = 6(x^2+x-2) = 6(x+2)(x-1)$.

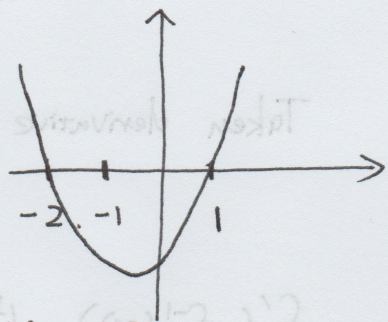
$f'(x)=0 \Leftrightarrow x=-2$ or $x=1$.

So at point $(-2, f(-2))$ $(1, f(1))$ the tangent line is horizontal.

b), c) $f(x)$ is increasing (resp. decreasing) if

$f'(x)>0$ (resp. $f'(x)<0$).

the graph of $f'(x)$ is



So. $f'(x)>0$ when

x is in $(-\infty,-2)\cup(1,+\infty)$

and $f'(x)<0$ when

x is in $(-2,1)$.

So. f is increasing when x is in $(-\infty,-2)\cup(1,+\infty)$.

f decreasing when x is in $(-2,1)$

d). tangent line. parallel to $y = -12x$ ~~when~~ when

$$f'(x) = -12$$

$$\Rightarrow \cancel{2x}\ 6x^2 - 6x - 12 = -12$$

$$\Leftrightarrow \quad 6x^2 - 6x = 0$$

then $x = 1$ or $x = 0$.

So at point $(1, f(1))$ and $(0, f(0))$

the tangent line is parallel to $y = -12x$

2. Ans: we have $f(f^{-1}(x)) = x$.

Taken derivative on both side, we have

(Using chain rule on the LHS)

$$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1.$$

Let $x = 2$. , ~~when~~ ~~whe~~ we have ~~$f^{-1}(2) = 1$~~

~~because $f(1) = 2$~~. $f'(f^{-1}(2))(f^{-1})'(2) = 1$.

~~$f'(f^{-1}(2)) = f'($~~ and we have $f^{-1}(2) = 1$ because $f(1) = 2$

So $f'(f^{-1}(2)) = f'(1) = 3 \Rightarrow (f^{-1})'(2) = \frac{1}{3}$

3 Ans. Using the fact $(\ln|x|)' = \frac{1}{x}$

we have

$$f'(x) = \frac{2\cos 2x \cdot 2}{2\sin 2x} = 2\cot 2x.$$

4. Ans: a) we have $g'(x) = f'(x)\tan x + f(x) \sec^2 x$.

So. $g'(\frac{4\pi}{3}) = f'(\frac{4\pi}{3})\tan\frac{4\pi}{3} + f(\frac{4\pi}{3})\sec^2\frac{4\pi}{3}$.

we have $f'(\frac{4\pi}{3}) = -7$ $\quad f(\frac{4\pi}{3}) = 2$.

$\tan\frac{4\pi}{3} = \sqrt{3}$. $\quad \cos\frac{4\pi}{3} = -\frac{1}{2}$

$\sec^2\frac{4\pi}{3} = \frac{1}{4}$.

$$g'(\frac{4\pi}{3}) = -7\cdot\sqrt{3} + \frac{1}{2}$$

b) we have $h'(x) = \frac{\sec x \tan x\, f(x) - f'(x)\sec x}{(f(x))^2}$.

$$h'(\frac{4\pi}{3}) = \frac{\sec\frac{4\pi}{3}\tan\frac{4\pi}{3} f(\frac{4\pi}{3}) - f'(\frac{4\pi}{3})\sec\frac{4\pi}{3}}{(f(\frac{4\pi}{3}))^2}$$

$$= \frac{-2\times\sqrt{3}\times 2 - (-7)\times(-2)}{2^2}$$

$$= \frac{-4\sqrt{3} - 14}{4} = -\frac{2\sqrt{3}+7}{2}.$$

5. Using the fact that $(\log_4 x)' = \dfrac{1}{\ln 4 \cdot x}$

So $$f'(x) = \frac{3x^2}{\ln 4 \; x^3} = \frac{3}{\ln 4 \cdot x}.$$

6. We have

$$y + xy' = \frac{16yy' - \cos x.}{1 + (8y^2 - \sin x)^2.}$$

Let $(x, y) = (\frac{\pi}{2}, \frac{1}{2})$.

$$\cos\frac{\pi}{2} = 0 \qquad \sin\frac{\pi}{2} = 1.$$

$$\frac{1}{2} + \frac{\pi}{2}y' = \frac{16 \times \frac{1}{2} y' - 0}{1 + (2-1)^2}$$

That is $\frac{1}{2} + \frac{\pi}{2}y' = 4y'$

$$1 + \pi y' = 8y'$$

$$y' = \frac{1}{8 - \pi}$$

7. a) $$\lim_{x\to 0} \frac{\tan 3x}{\sin 4x} = \lim_{x\to 0} \frac{\sin 3x}{\cos 3x \,\sin 4x}.$$

$$= \lim_{x\to 0} \frac{\sin 3x}{\sin 4x} \cdot \lim_{x\to 0} \frac{1}{\cos 3x}.$$

we have $\lim_{x\to 0} \cos x = \cancel{4}1$ So $\lim_{x\to 0} \frac{1}{\cos 3x} = 1$.

and $$\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} = \lim_{x\to 0} \frac{3\sin 3x}{3x} \lim_{x\to 0} \frac{4x}{4\sin 4x}$$

$$= 3 \times \frac{1}{4}. = \frac{3}{4}.$$

(~~lim~~ b) using the fact ~~sin t~~ $\cos x = \sin\left(\frac{\pi}{2} - x\right)$.

$$\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} = \lim_{x\to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\frac{\pi}{2} - x}.$$

Let $u = \frac{\pi}{2} - x$ $$\lim_{u\to 0} \frac{\sin u}{u} = 1.$$

8. we have $\ln y = \ln x^{\sinh x} = \sinh x \ln x$.

Taking der. we have

$$\frac{y'}{y} = \cosh x \ln x + \sinh x \cdot \frac{1}{x}$$

$$y' = y\left(\cosh x \ln x + \sinh x \frac{1}{x}\right) = x^{\sinh x}\left(\cosh x \ln x + \frac{\sinh x}{x}\right)$$

9. $\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$$\sinh x \cosh x = \frac{e^x - e^{-x}}{2} \cdot \frac{e^x + e^{-x}}{2}$$

$$= \frac{1}{4}(e^x - e^{-x})(e^x + e^{-x}).$$

Using $(a-b)(a+b) = a^2 - b^2$.

Let $a = e^x \quad b = e^{-x}$.

$$\sinh x \cosh x = \frac{1}{4}\left((e^x)^2 - (e^{-x})^2\right) = \frac{1}{4}\left(e^{2x} - e^{-2x}\right).$$

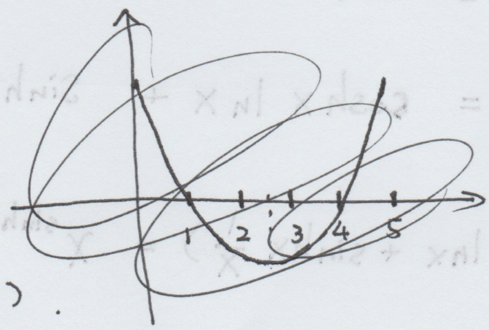
$$= \frac{1}{2}\left(\frac{e^{2x} - e^{-2x}}{2}\right)$$

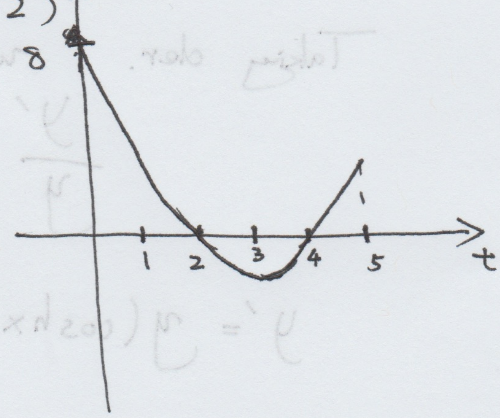
$$= \frac{1}{2}\sinh 2x.$$

$$\Rightarrow \frac{\sinh x \cosh x}{\sinh 2x} = \frac{1}{2}.$$

10 Ans. $v(t) = s'(t) = t^2 - 6t + 8 = (t-4)(t-2)$

$v(t)$ looks like





So slowing up when

t is in $(0,2) \cup (3,4)$.

11.

$$G(t) = G(0)\, e^{kt}.$$

where $G(0) = 100$

and $G(2) = 50 = G(0)\, e^{2k} = 100\, e^{2k}$

So $e^{2k} = \frac{1}{2}$ and $2k = \ln\frac{1}{2} = -\ln 2$.

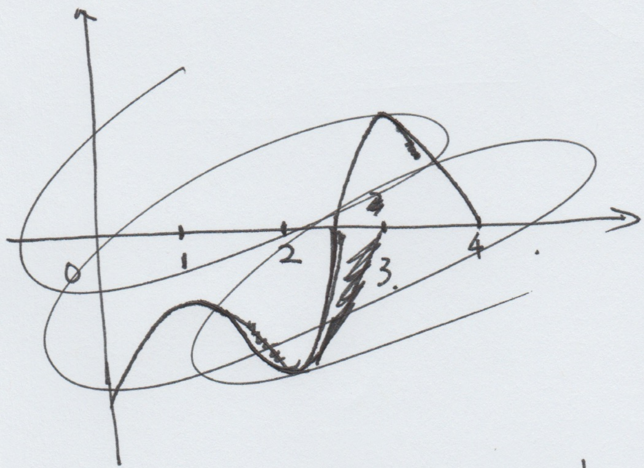
$$k = -\frac{\ln 2}{2}.$$

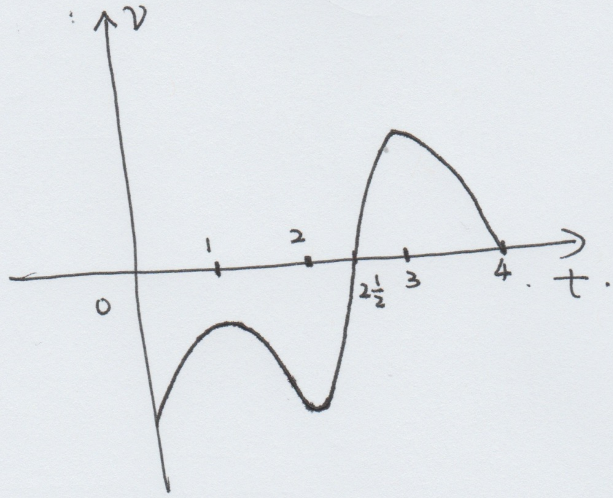
So $G(t) = G(0)\, e^{-\frac{\ln 2}{2}t} = 100\, \left(e^{\ln 2}\right)^{-\frac{t}{2}}$.

$$= 100\ 2^{-\frac{t}{2}}$$

$$G(3) = 100 \cdot 2^{-\frac{3}{2}} = 100 \cdot e^{-\frac{3}{2}\ln 2}.$$

12. $v - t$ relation will be





Speeding up when t is in $(1, 2) \cup (\frac{5}{2}, 3)$.