

In the last class we gave an excepte

If a nonaffine scheme defined by

gluing. We want to generalize this

excepte, but it's better to use a

different construction, which we now

explain.

En The polynomial ring R: SCX, -- x)

nite the standard grading is

of a gradd ring.

Gim a graded rig R,

R, = R, & R 2 & ....
is an ided collect he irratevent ided

Thm/Def Given a graled ring

R. Lob Proj R = { homog. prime i'dels

pt & P C R }

This can be made into a schen when the basic open subscene D(f) = { Pe Proj. R | f x P} who f homog. of deg d

Proje (D(f)) = DR Rkd. Tk

= (RC-f)

this has a

natural 7-grading

A pf can be found in Hartshorns

PP 76-77. It kay point

is  $\{D(f) = Spec(REM_f)\}$ a covering by affine scheme.

(Note Hartshorn use, D(f) which conflicts with affine of the conflicts with affine of the conflicts of the conflict of the conflicts of the confl

whe S= k is an alg. closed frield

the closed pt, of the new An and IP's

will classical affirm or projection

space i.e pts of k or lines in know

So in this sense the new and

old versions are the 'sam'

In general, set

U. = D(x.) = Spec S(xo -- xo)

R

= A's

## 2 Closed Subschemes



Del Amorphism (f, fy): (XO) -> (You)

I achous is simply a marphism of

locally ringed spaces

Del A morphism (,,, , ): (x, ox) ~, (y ox)

of scheme, is colled an closed immersion

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of i: X ~> y is homeomorphism between

X and a closed subsche

(in the sheef some). We may as

well tale X = i (x) and we reform

by (x, ox) are a closed subsche

of y.

ex If I CR is on idd, the
equation to map R-3 R/I indus,
a closed, unesses spec (R/Z) -> Speck.

All closed immersions are locally of
this form.

Det Aprojective schen on R

is a closed subschen of some

IP?

3 Coherent Sheaves on Schen

Def (f (X,R) is a ringed spece, a shed

of R-mobble M is quasicoherent

of every pt re X has a while U sit

of Out -> Out -> out (I, I may

be infinite)

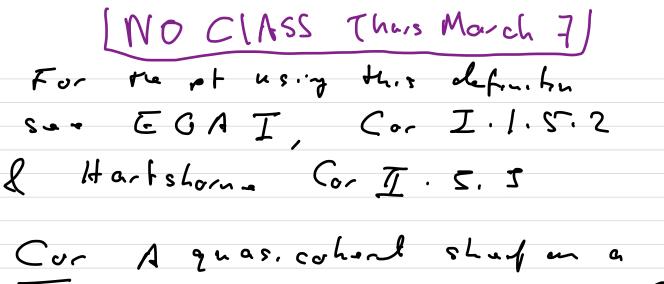
The If Risaring, and Misan R-module, Ken

M(u)=M& C (u)

definer a quesicolint short Speck.

This gives an equivalence of calogoniss

(R-mod) ~ (Sheams a Speck)



(C)

Cor Aquas, cohond shaff on a chan locally of the form M,

Jet Aschen is noetherica of it has finite open by schen of Ke form { u.= Speak, i > nih R. noetheria.

All the school in this class will have this property so the condide shall be assued.

Def If X is a noetherian schene,

a quasicoherent sheef M,; coherent

If , I is locally of the form M.

with M finifly generald.

Next is a fundamental excel.

(== 1 fi: (Y O) -> (x O) ... a

closed entschen of a (nontheria) schen

Then cl = Kar (Ox -> in e) y

is collect the ideal sheef of x. This
is coherent.

A suberale.

Ex let y C X be closed then let

cl (u) = {fe ex (u) | f| = 0}.

This makes ey, into a veduced

shad, of vinges is of 0 (u) has no

nilpotente This is called the

veduced quischeme structor of y

The complement of Uo CIP define,

a closul got H = Vip (xu).

The reduced subschene structure make,

It S IP<sup>N-1</sup> The isled when f will be

analyzed below. Bet first

The Det Given a noethern

Viz Sant a fin. gen Z-graded mobbe

Mover R=SZxo, --xn),

there exists a coherent sheet M

one IP's such that

 $\tilde{m}$  ( D(f)) =  $(M[\frac{1}{f}])$ for f homog. This, gives a event f u-ctor

(figgrald) = (coherent shame P)

Pt See Hartshorn pp 116-117 & EGAII 2.5 (They just write M)

For example = (S/S+) = 0

Given an gradel R-mod M

let M(d) = M as a module both

now grading

M(d) = M

Mre

Def Opi (d) = S(d)

To (1) is called the fautological

"line built " or invortible sheet

(We explain what this means later.)

Previous Ex (cont) The isled should

If H C IP is isomorphis to O(-1)

IP

To see this, observe d= J, wh

I = (x3) = S(-1) since it; the inex

A S (-1) x3

Ex Mora querelly if / CIP is a closell subschen lefinal by a deg l homon poly; if rile ly a deg l homon poly; if r

Det A coherent shuf Mon a schem X
i's locally free of rack vif Ja open offin
con {U.i } ste. Mlu. = Mi, who Mi i's france
of rout v