ON THE TRACY-WIDOM$_\beta$ DISTRIBUTION FOR $\beta = 6$.

Abstract In this talk, we will present some recent progress in the rigorous asymptotics analysis of the Tracy-Widom distribution function for Dyson's β - ensemble with $\beta = 6$. Specifically, we will discuss the recent work of I. Rumanov on a Lax - pair representation for the Bloemendal-Virag equation describing the Tracy-Widom functions corresponding to general values of β. In particular, we will intepret the Rumanov Lax-pair as a gauge transform of the Frlachska- Newell Lax pair for the second Painlevé equation (the work of T. Grava, A. Kapaev, F. Mezzadri, and the speaker). We shall also reflect on the most recent paper of M. Bertola, M. Cafasso, and V. Rubsov, where they prove the Lax-pair integrability of the Calogero-Painlevé systems. Since Rumanov's work can be also interpreted as a reduction of the Bloemendal-Virag equation to one of the six Calogero-Painlevé systems, the Bertola-Cafasso-Rubsov result makes it possible to develop a rigorous approach to the tail asymptotics to the $\beta = 6$ Tracy-Widom distribution function and, in fact, to the all even values of beta.