

An introduction to DAHA superpolynomials
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The Khovanov-Rozansky stable reduced polynomial, the most advanced (numerical) knot invariants, found many applications in mathematics, including Hilbert schemes of plane curve singularities and those of some surfaces. Their physics counterparts, called superpolynomials, occur in the theory of BPS states and M_5 -theory. The interpretation (and calculation) of HOMFLY-PT polynomials of torus knots via CFT and Verlinde algebras was extended recently to the theory of DAHA superpolynomials. They can be defined so far only for torus iterated links (including all algebraic links), but there are no problems with links, any colors (Young diagrams), and the reduced setting in contrast to the Khovanov-Rozansky polynomials, which are mostly restricted to uncolored knots and are very difficult to calculate even for torus knots. The most advanced text by now is "DAHA approach to iterated torus links" (with I. Danilenko). One of the applications here is the topological DAHA-vertex, based on DAHA superpolynomials for Hopf links, with applications to refined Rogers-Ramanujan series and beyond.

From the physics perspective, the relation of DAHA superpolynomials to the geometric superpolynomials for plane curve singularities (which require only the singularity for their definition) seems related to the Landau-Ginzburg Sigma Models for superpotentials W (equations of singularities). The program in Vafa-Warner's paper "Catastrophes..." (1989) was to study Field Theories associated with LGSM as directly as possible in terms of W ; geometric superpolynomials do exactly this! Namely, the DAHA superpolynomials are clearly certain correlation (n -point) functions of refined knot operators and their products. So modulo their conjectural connection with the geometric superpolynomials, the latter provide advanced information about the corresponding physics model directly in terms of W .

A possible connection with LGSM will be touched upon if time permits. The plan is to give the definition of DAHA of type A_1 , calculate the corresponding refined Jones polynomial and superpolynomial for trefoil, and then explain general constructions and connections.