MA 158

Quiz 15 $\qquad$ 14 $No\epsilon\mu\beta\rho\iota o\varsigma$ 2016

Instructions: Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem. (4 points) For the system of equations

$$\begin{cases} x^2 + y^2 = 8 \\ y - x = k \end{cases}$$

find all values of k so that there will be

(a) one solution,

(b) two solutions, and

(c) no solutions

Solution. Solving the second equation for y, we have $y = x + k$, and we plug that in to the first equation. This gives

$$\begin{aligned} x^2 + y^2 &= 8 \\ x^2 + (x+k)^2 &= 8 \\ x^2 + x^2 + 2kx + k^2 &= 8 \\ 2x^2 + 2kx + k^2 - 8 &= 0. \end{aligned}$$

With $a = 2,\ b = 2k,\ c = k^2 - 8$, we compute the discriminant:

$$\begin{aligned} \Delta = b^2 - 4ac &= (2k)^2 - 4(2)(k^2 - 8) \\ &= 4k^2 - 8k^2 + 64 \\ &= -4k^2 + 64. \end{aligned}$$

Now this amounts to finding things about Δ. For (a), there is one solution when $\Delta = 0$, for (b), we will get two solutions when $\Delta > 0$ and for (c) we will have no solutions when $\Delta < 0$.

Setting $\Delta = 0$,

$$\begin{aligned} 0 &= -4k^2 + 64 \\ &= -4(k^2 - 4) \\ &= -4(k+4)(k-4), \end{aligned}$$

which gives $k = \pm 4$. Using a number line we see that if $k < -4$ or $k > -4$ then $\Delta < 0$ and when $-4 < k < 4$, then $\Delta > 0$.

$$\begin{array}{ccccc} \ominus & & \oplus & & \ominus \\ \hline & -4 & & 4 & \end{array}$$

To conclude, we have

(a) one solution when $k = \pm 4$,

(b) two solutions when k is in the interval $(-4, 4)$, and

(c) no solutions when k is in $(-\infty, -4) \cup (4, \infty)$. ☺