MA 158

31 August 2016

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Instructions: Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (4 points) Rationalize the roots in the following expression. Simplify completely.

$$\frac{2-x}{\sqrt{x}+\sqrt{2}}$$

Solution. Remember, any time we rationalize roots, we should be thinking about multiplying by the conjugate. The conjugate of $\sqrt{x} + \sqrt{2}$ is $\sqrt{x} - \sqrt{2}$, and

$$(\sqrt{x} + \sqrt{2})(\sqrt{x} - \sqrt{2}) = x - \sqrt{x}\sqrt{2} + \sqrt{2}\sqrt{x} - 2$$

= x - 2. (*)

Now multiplying the top and bottom of the fraction by $\sqrt{x} - \sqrt{2}$,

$$\frac{2-x}{\sqrt{x}+\sqrt{2}} \cdot \frac{\sqrt{x}-\sqrt{2}}{\sqrt{x}-\sqrt{2}} \stackrel{(*)}{=} \frac{(2-x)(\sqrt{x}-\sqrt{2})}{x-2}$$
$$= \frac{(2-x)(\sqrt{x}-\sqrt{2})}{-(2-x)} = \frac{\sqrt{x}-\sqrt{2}}{-1}$$
$$= -(\sqrt{x}-\sqrt{2}) = \sqrt{2} - \sqrt{x}. \quad (**)$$

Either of the answers in (**) are perfectly acceptable.

Note here that an answer of

$$\frac{(2-x)(\sqrt{x}-\sqrt{2})}{x-2}$$

is incomplete as it is not simplified *completely*. To receive full points it was crucial to recognize that 2 - x = -(x - 2) and use that information to allow for some cancellation. A common mistake was to write

$$\frac{(2-x)(\sqrt{x}-\sqrt{2})}{x-2} = \frac{(-1)(2-x)(\sqrt{x}-\sqrt{2})}{x-2}.$$

Quiz 1

It is important to understand what we are doing as we are trying to cancel out things from the top and the bottom. Remember that we can only multiply by some version of 1 to preserve equality. So more explicitly, we could write

$$\frac{(2-x)(\sqrt{x}-\sqrt{2})}{x-2} = \frac{(-1)(-1)(2-x)(\sqrt{x}-\sqrt{2})}{x-2},$$

and now we distribute one of the (-1)'s over the (2 - x) in the numerator. And we get

$$\frac{(-1)(-1)(2-x)(\sqrt{x}-\sqrt{2})}{x-2} = \frac{(-1)(-2+x)(\sqrt{x}-\sqrt{2})}{x-2}$$

Another common mistake was to "distribute" -1 by multiplying it by both 2 - x and $\sqrt{x} - \sqrt{2}$. By this I mean, some students wrote

$$(-1)\frac{(2-x)(\sqrt{x}-\sqrt{2})}{x-2} = \frac{(-2+x)(-\sqrt{x}+\sqrt{2})}{x-2}.$$

You'll notice that when you do this you end up with the correct answer, but this is because you actually multiplied by (-1)(-1). The take away here is to remember that the distributive property deals with multiplication over addition, i.e.,

$$a(b+c) = ab + ac.$$

However, if we consider a triple product abc, we can write

$$abc = (ab)c = a(bc).$$

This is called associativity. But it is not true that a(bc) = (ab)(ac). Try it out with any numbers other than 1 if it is still unclear.