MA 158

Quiz 2 — 7 septiembre 2016

Instructions: Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (2 points) Given

$$f(x) = \frac{7x+1}{9-x}, \quad g(x) = \frac{1}{x},$$

find $(f \circ g)(x)$.

Solution.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{7\left(\frac{1}{x}\right)+1}{9-\frac{1}{x}}$$ ☺

Problem 2. (2 points) Find $(g \circ f)(x)$ with the same f and g.

Solution.

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{7x+1}{9-x}\right) = \frac{1}{\frac{7x+1}{9-x}} = \frac{9-x}{7x+1}$$ ☺

Problem 3. (Bonus 1 point) Find the domain of $f \circ g$ and $g \circ f$.

Solution. Recall that for x to be in the domain of $f \circ g$, we need x to be in the domain of g and $g(x)$ to be in the domain of f. In other words, we need to check what values of x we can and can't plug into g, and then check what values of $g(x)$ we can and can't plug into f.

For $f \circ g$, looking at $g(x)$, we can't have $x = 0$. And for $f(g(x))$, we can't have $9 - \frac{1}{x} = 0$. Multiplying both sides of the equation by x, we see

$$\begin{aligned} 9 - \frac{1}{x} &= 0 \\ 9x - 1 &= 0 \\ 9x &= 1 \\ x &= \frac{1}{9}. \end{aligned}$$

So, combined with the fact that we can't have $x = 0$, the domain of $f \circ g$ is $(-\infty, 0) \cup (0, 1/9) \cup (1/9, \infty)$.

Now for $g \circ f$, we can't have $9 - x = 0$, i.e., we can't have $x = 9$. Then for $g(f(x))$, we can't have $7x + 1 = 0$, i.e., $x = -\frac{1}{7}$. Putting these two facts together, we see that the domain for $g \circ f$ is $(-\infty, -1/7) \cup (-1/7, 9) \cup (9, \infty)$. ☺

In general when we're looking at a composition $f \circ g$, students usually forget to check whether x is in the domain for g. This usually happens because once they plug in $g(x)$ into f and simplify, information gets lost. For example we could rewrite $f \circ g$ in Problem 1 as

$$(f \circ g)(x) = \frac{7 + x}{9x - 1}$$

after multiplying by $\frac{x}{x}$. As a standalone function, the domain of

$$\frac{7 + x}{9x - 1}$$

is $(-\infty, 1/9) \cup (1/9, \infty)$. But as the composition of f and g, we still can't have $x = 0$ because we are not allowed to plug 0 into $g(x)$, so $(f \circ g)(0) = f(g(0))$ is undefined.