Quiz 4   26 setembro 2016

Instructions: Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (4 points) Find an equation for a polynomial of degree 5 with the following properties:

1. zeros at $x = 1$ and $x = -7$;

2. $f(0) = -16$;

3. $f(x) > 0$ only on the interval $(-\infty, -7)$.

Solution. Zeros at $x = 1$ and $x = -7$ means f has the form

$$f(x) = a(x - 1)(x + 7).$$

We use property 3 to figure out multiplicities. $f(x) > 0$ on $(-\infty, -7)$ and $f(x) < 0$ on $(-7, 1) \cup (1, \infty)$ means that we have a sign change at $x = -7$ but not at $x = 1$. So $x = -7$ has odd multiplicity and $x = 1$ has even multiplicity. So our options are

$$f(x) = a(x + 7)(x - 1)^4 \tag{1}$$
$$f(x) = a(x + 7)^3(x - 1)^2. \tag{2}$$

Using $f(0) = -16$, in (1), we get

$$f(0) = -16 = a(7)(-1)^4 \Rightarrow a = \frac{-16}{7},$$

and in (2) we get $f(0) = -16 = a(7)^3(-1)^2 \Rightarrow a = \frac{-16}{7^3}$. So our final answer is

$$f(x) = \frac{-16}{7}(x + 7)(x - 1)^4 \quad \text{or}$$
$$f(x) = \frac{-16}{7^3}(x + 7)^3(x - 1)^2. \qquad \qquad \text{☺}$$