MA 158

Quiz 5 | 27 setembre 2016

Instructions: Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (4 points) Given the following rational function

$$f(x) = \frac{4x+1}{5x^2+55x+90},$$

find the following. If it does not exist, say so.

(a) The zeros of $f(x)$

(b) Vertical asymptotes

(c) Horizontal asymptotes

(d) Point at which the graph crosses the horizontal asymptote

Solution.

(a) For a rational function $f(x) = \frac{p(x)}{q(x)}$, the zeros occur when $p(x) = 0$. So the zeros occur when $4x + 1 = 0$. That is, $x = -1/4$.

(b) Vertical asymptotes occur when $q(x) = 0$. So we set $5x^2 + 55x + 90 = 0$. So,

$$\begin{aligned} 5x^2 + 55x + 90 &= 5(x^2 + 11x + 18) \\ &= 5(x+2)(x+9) = 0. \end{aligned}$$

And we see that the vertical asymptotes occur at $x = -2$ and $x = -9$.

(c) Here we examine $\deg p(x)$ vs. $\deg q(x)$. In this case, we have $\deg q(x) > \deg p(x)$, and when this occurs, we have a horizontal asymptote of $y = 0$.

(d) To find the point where the graph crosses the horizontal asymptote, we set $f(x) = HA$. So we have

$$f(x) = \frac{4x + 1}{5x^2 + 55x + 90} = 0,$$

cross multiplying, we get $4x + 1 = 0$, or $x = -1/4$. So the only point where this happens is $(-1/4, 0)$.

Alternatively, we could have recognized that since our horizontal asymptote is 0, the points where the graph crosses the horizontal asymptote are simply the zeros of f. ☺