MA 16020

Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (7 points) Find and classify all critical points of the function

$$f(x,y) = 54x^4 + 64x + \frac{16}{3}y^3 - y + 2.$$

Solution. We start by taking the first partial derivatives:

$$f_x = 216x^3 + 64$$
$$f_y = 16y^2 - 1.$$

Setting $f_x = 0$ gives

$$216x^{3} + 64 = 0$$

$$216x^{3} = -64$$

$$x^{3} = \frac{-64}{216}$$

$$x = -\frac{4}{6}$$

$$= -\frac{2}{3}$$

And setting $f_y = 0$ gives

$$16y^{2} - 1 = 0$$
$$16y^{2} = 1$$
$$y^{2} = \frac{1}{16}$$
$$y = \pm \frac{1}{4}$$

0

Thus we have critical points at (-2/3, 1/4) and (-2/3, -1/4). Calculating the second partial derivatives,

$$f_{xx} = 648x^2$$
$$f_{yy} = 32y$$
$$f_{xy} = 0$$

Then the discriminant is

$$D = (648x^2)(32y) - 0.$$

At the point (-2/3, 1/4) we have $D = (648(-2/3)^2)(32(1/4)) > 0$ and $f_{xx} = 648(-2/3)^2 > 0$, which gives us a local min. And at the point (-2/3, -1/4), we have $D = (648(-2/3)^2)(32(-1/4)) < 0$, which tells us that this is a saddle point.

Problem 2. (3 points) When is our next exam?

Solution. Our next exam is Monday.