Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points. **Give an exact answer unless otherwise noted.**

Problem 1. (5 points) Evaluate

$$\int_2^7 \frac{1}{1-t}\,dt.$$

Solution.

$$\begin{aligned}
\int_2^7 \frac{1}{1-t}\,dt &= -\int_{t=2}^{t=7} \frac{du}{u} \\
&= -\int_{-1}^{-6} \frac{du}{u} \\
&= -\ln|u|\Big|_{-1}^{-6} \\
&= -(\ln|-6| - \ln|-1|) \\
&= -\ln 6 + \ln 1 \\
&= -\ln 6
\end{aligned}$$

$$\begin{aligned}
u &= 1-t \\
du &= -dt \\
-du &= dt \\
t = 2 \Rightarrow u &= 1-2 = -1 \\
t = 7 \Rightarrow u &= 1-7 = -6
\end{aligned}$$

□

Problem 2. (5 points) It is estimated that t weeks from now the average price of a gallon of gas will be increasing at a rate of

$$p'(t) = \frac{t}{t^2+19}.$$

If the average price of a gallon of gas is \$2.13, what will the average price of a gallon of gas be 9 weeks from now? Give your answer to the nearest cent.

Solution. Even though we are looking for the average price of a gallon of gas, we are not looking for the average value of the price function. The problem is suggesting that $p(t)$ gives us the average price of gas at time t weeks from now. So our goal is just to find the value $p(9)$.

$$\begin{aligned}
p(t) &= \int \frac{t}{t^2+19}\,dt \\
&= \frac{1}{2}\int \frac{du}{u} \\
&= \frac{1}{2}\ln|u| + C \\
&= \frac{1}{2}\ln(t^2+19) + C.
\end{aligned}$$

$$\begin{aligned}
u &= t^2+19 \\
du &= 2t\,dt
\end{aligned}$$

Using $p(0) = 2.13$, we have

$$\begin{aligned} 2.13 &= \frac{1}{2}\ln(0^2+19)+C \\ 2.13 &= \frac{1}{2}\ln(19)+C \\ C &= 2.13 - \frac{1}{2}\ln(19). \end{aligned}$$

So

$$\begin{aligned} p(9) &= \frac{1}{2}\ln(9^2+19)+2.13-\frac{1}{2}\ln(19) \\ &\approx \$2.96. \end{aligned}$$

□

Problem 3. (Up to 1/0 points) Approximate e to 15 decimal place accuracy.

Solution. $e \approx 2.718281828459045$. □