Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (4 points) Determine whether the following differential equations is linear.

(a.) $y' + (x^2 + \cos x)y = x$

(b.) $yy' = x$

(c.) $y' + xy^2 = x - 1$

(d.) $y' - xy = x^2 y + 3$

Solution. (a) Yes

(b) No

(c) No

(d) Yes □

Problem 2. (4 points) Find the general solution for the differential equation

$$xy' + y = 3y' + x^2 + 5, \qquad x > 3.$$

Solution. This is a first-order linear equation. So the first thing we want to do is combine all the y's and then get it in the desired form.

$$\begin{aligned} xy' + y &= 3y' + x^2 + 5 \\ xy' - 3y' + y &= x^2 + 5 \\ (x-3)y' + y &= x^2 + 5 \\ y' + \frac{1}{x-3}y &= \frac{x^2+5}{x-3}. \end{aligned}$$

Next we want to find our integrating factor

$$\begin{aligned} \mu(x) &= e^{\int \frac{1}{x-3}\,dx} \\ &= e^{\ln(x-3)} \\ &- x - 3. \end{aligned}$$

Notice that we don't need absolute values on the $\ln(x-3)$ since we're working

with $x > 3$. Now

$$\begin{aligned} y\mu(x) &= \int \mu(x)q(x)\,dx + C \\ y(x-3) &= \int (x-3)\frac{x^2+5}{x-3}\,dx + C \\ y(x-3) &= \int \left(x^2+5\right)\,dx + C \\ y(x-3) &= \frac{1}{3}x^3 + 5x + C \\ y &= \frac{x^3}{3(x-3)} + \frac{5x}{x-3} + \frac{C}{x-3}. \end{aligned}$$

□

Problem 3. (2 points) When is our next exam on February 20?

Solution. The answer is in the question. □