Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (4 points each) Determine whether the following series converge or diverge. State why or why not. If the series converges, compute the sum.

(a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{3}{2}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^{n+1}}$

Solution. (a) This diverges since $r = \frac{3}{2} > 1$.

(b) This converges since $r = \frac{1}{2} < 1$. Using the given formula,

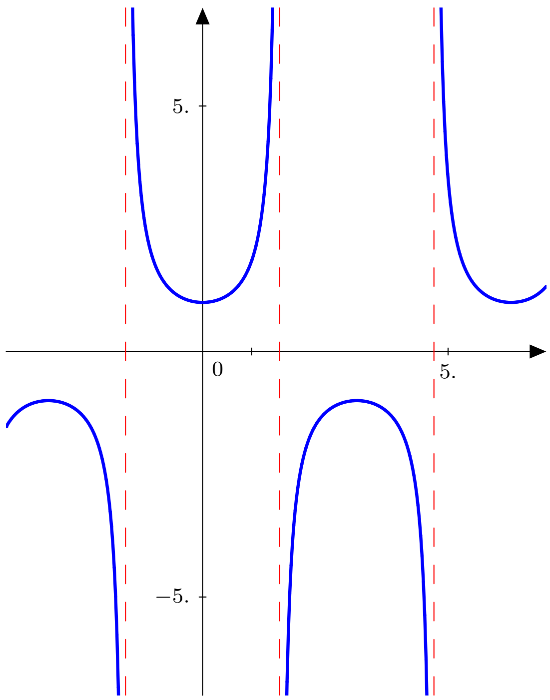
$$\begin{aligned}\sum_{n=1}^{\infty}\frac{1}{2^{n+1}} &= \sum_{n=0}^{\infty}\frac{1}{2^{n+2}}\\ &= \sum_{n=0}^{\infty}\frac{1}{4}\frac{1}{2^n}\\ &= \frac{1}{4}\cdot\frac{1}{1-\frac{1}{2}}\\ &= \frac{1}{4}\cdot 2\\ &= \frac{1}{2}.\end{aligned}$$

□

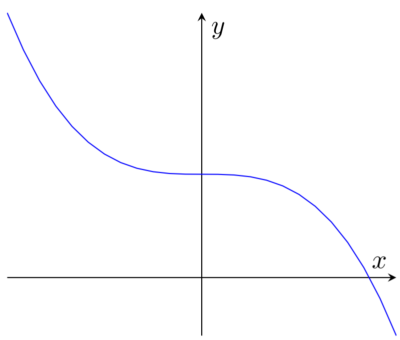
Problem 2. (2 points) Sketch a graph of any two of the following functions.

1. $y = \sec x,\ 0 \le x \le 2\pi$
2. $y = -x^3 + 8$
3. $y = \sqrt{4 - x^2}$
4. $y = 4\ln x$

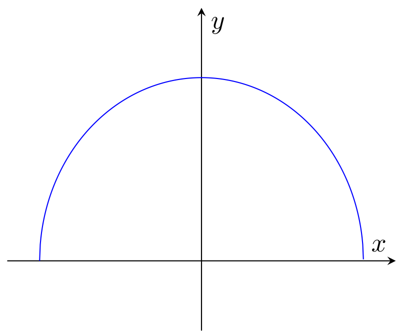
Solution. 1. The graph of secant is below. The pattern continues, but you just needed to keep it between 0 and 2π.



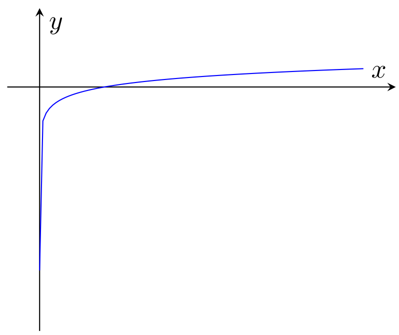
2. Graphing $-x^3 + 8$ should be straightforward.



3. It's true that squaring both sides gives us $x^2 + y^2 = 4$, which is a circle. But since this is a positive square root, this is just the top half of the circle.



4. It's important to remember that $y = 4\ln x$ has domain $(0, \infty)$. Moreover, the x-intercept is 1 since $\ln 1 = 0$.



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Recall. If $|r| < 1$, then
$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$