MA 16020

Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (6 points) For the given function and values of z

$$f(x,y) = \ln(y - e^{5x}), \quad z = 0, \ln 10,$$

- (a) What is the domain of this function?
- (b) What type of function describes the level curves?
- (c) Give a sketch of the level curves.
- (d) What functions y = f(x) do you get for these values of z?

Solution. (a) The argument of the logarithm must be positive, so we need $y - e^{5x} > 0$, or $y > e^{5x}$. In set notation, this is

$$\{(x,y): y > e^{5x}\}.$$

(b) The level curves are given by f(x,y) = k, where k is a constant. That is,

$$k = \ln(y - e^{5x})$$

$$e^{k} = y - e^{5x}$$

$$y = e^{5x} + e^{k},$$
(*)

which are exponential functions.

(c) For the given values of $z = 0, \ln 10$ the level curves look like



(d) For the given values of z, we just plug in k=0 and $k=\ln 10$ into (*) to get the functions

$$y = e^{5x} + 1$$
 and $y = e^{5x} + 10$.

Note that we used the fact that $e^0 = 1$ and $e^{\ln 10} = 10$.

Problem 2. (2 points) What is the domain of the following function?

$$f(x,y) = \ln\left(x^2 + y^2\right)$$

Solution. We just need $x^2 + y^2$ to be positive, which is always the case unless x = y = 0. So the domain is

$$\{(x,y): x^2 + y^2 \neq 0\}.$$

Problem 3. (2 points) Which lesson numbers will be relevant for our exam over lessons 13 through 19?

Solution. Lessons 13 through 19.