**Instructions.** Show all work, with clear logical steps. No work or hard-to-follow work will lose points. **Give an exact answer unless otherwise noted.**

**Problem 1.** (4 points) Evaluate

$$\int_2^7 \frac{1}{1-t}\,dt.$$

*Solution.*

$$\begin{aligned}
\int_2^7 \frac{1}{1-t}\,dt &= -\int_{t=2}^{t=7} \frac{du}{u} && u = 1-t \\
&= -\int_{-1}^{-6} \frac{du}{u} && du = -dt \\
&= -\ln|u|\Big|_{-1}^{-6} && -du = dt \\
&= -(\ln|-6| - \ln|-1|) && t=2 \Rightarrow u = 1-2 = -1 \\
&= -\ln 6 + \ln 1 && t=7 \Rightarrow u = 1-7 = -6 \\
&= -\ln 6
\end{aligned}$$

□

**Common mistakes.** The most common mistake was forgetting the negative sign when differentiating $u = 1 - t$. Another very common mistake was not noting that 2 and 7 were the bounds for $t$ after putting the integral in terms of $u$. It is fine to write

$$\int_{t=2}^{t=7} \frac{du}{u}$$

and put things back in terms of $t$ after computing the antiderivative. What you want to avoid is writing

$$\int_2^7 \frac{du}{u},$$

as that is unclear that you mean the bounds are in terms of $t$, and it is not technically equal to the original integral.

**Problem 2.** (5 points) It is estimated that $t$ weeks from now the average price of a gallon of gas will be increasing at a rate of

$$p'(t) = \frac{t}{t^2+19}.$$

If the average price of a gallon of gas is \$2.13, what will the average price of a gallon of gas be 9 weeks from now? Give your answer to the nearest cent.

*Solution.* Even though we are looking for the average price of a gallon of gas, we are not looking for the average value of the price function. The problem is

suggesting that $p(t)$ gives us the average price of gas at time $t$ weeks from now. So our goal is just to find the value $p(9)$.

$$\begin{aligned} p(t) &= \int \frac{t}{t^2+19}\,dt \\ &= \frac{1}{2}\int \frac{du}{u} && u = t^2+19 \\ &= \frac{1}{2}\ln|u| + C && du = 2t\,dt \\ &= \frac{1}{2}\ln(t^2+19) + C. \end{aligned}$$

Using $p(0) = 2.13$, we have

$$\begin{aligned} 2.13 &= \frac{1}{2}\ln(0^2+19) + C \\ 2.13 &= \frac{1}{2}\ln(19) + C \\ C &= 2.13 - \frac{1}{2}\ln(19). \end{aligned}$$

So

$$\begin{aligned} p(9) &= \frac{1}{2}\ln(9^2+19) + 2.13 - \frac{1}{2}\ln(19) \\ &\approx \$2.96. \end{aligned}$$

□

**Common mistakes.** The most common mistake was interpreting the word "average" as needing to use the average value formula which says that

$$f_{\text{avg}}[a,b] = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

But here, $p'(t)$ represents the rate of change of the "average price of gas," which means that $p(t)$ represents the average price of gas.

**Problem 3.** (1 point) What is the name of your instructor for MA 16020.