Quiz 4 **MA 16020** **Nick Egbert**

Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (5 points) Find a general solution to the given differential equation.

$$9x^2y' = y' + 9xe^{-y}$$

Solution.

$$\begin{aligned}
9x^2y' &= y' + 9xe^{-y} \\
9x^2y' - y' &= 9xe^{-y} \\
(9x^2-1)y' &= 9xe^{-y} \\
y' &= 9xe^{-y} \qquad\qquad y' = \frac{dy}{dx} \\
e^y\,dy &= \frac{9x}{9x^2-1}\,dx \\
\int e^y\,dy &= \int \frac{9x}{9x^2-1}\,dx
\end{aligned}$$

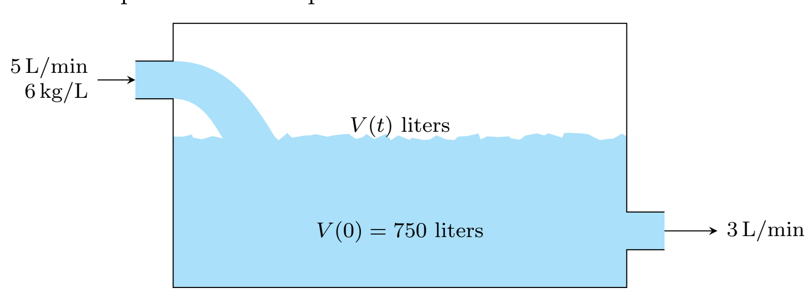
$$\begin{aligned}
u &= 9x^2 - 1 \\
du &= 18x\,dx \\
\frac{1}{2}\,du &= 9x\,dx
\end{aligned}$$

$$\begin{aligned}
e^y &= \frac{1}{2}\int \frac{du}{u} \\
e^y &= \frac{1}{2}\ln|u| + C \\
e^y &= \frac{1}{2}\ln\left|9x^2-1\right| + C \\
y &= \ln\left(\frac{1}{2}\ln\left|9x^2-1\right| + C\right)
\end{aligned}$$

$\square$

Problem 2. (4 points) A 1000-liter tank initially contains 750 liters of brine containing 50 kilograms of dissolved salt. Brine containing 6 kilograms of salt per liter flows into the tank at the rate of 5 liters per minute, and the well-stirred mixture flows out of the tank a rate of 3 liters per minute. **Draw a picture that illustrates this scenario and set up a differential equation for the amount of salt in the tank at time t.**

Solution. The picture below represents the situation we're after.



To determine the differential equation, we let $A(t)$ represent the amount of salt in the tank at time t. Then

$$\begin{aligned}\frac{dA}{dt} &= (\text{Rate in}) - (\text{Rate out})\\ &= \frac{5\text{ L}}{\text{min}} \cdot \frac{6\text{ kg}}{\text{L}} - \frac{3\text{ L}}{\text{min}} \cdot \frac{A(t)\text{ kg}}{V(t)\text{ L}}\\ &= \left(30 - \frac{3A(t)}{V(t)}\right)\frac{\text{kg}}{\text{min}}.\end{aligned}$$

So we just need to figure out $V(t)$. But $dV/dt = 5-3 = 2$. So $V(t) = 2t+V(0) = 2t + 750$. So the differential equation is

$$\frac{dA}{dt} = 30 - \frac{3A}{2t+750}. \qquad \square$$

Problem 3. (1 point) How is the course going? What could be better? What's going well?