Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (4 points) Determine whether the following integral converges or diverges. If it converges, compute it.

$$\int_2^\infty \frac{1}{x(\ln x)^{3/2}}.$$

Remark. For the 11:30 class, there was a typo on this problem, with the integral starting at 1, which gives us a problem since $\frac{1}{x(\ln x)^{3/2}}$ is discontinuous at $x = 1$. If you did the integral without worrying about this, you received full points.

Solution. We start by making a u-substitution with $u = \ln x$ so that $du = \frac{1}{x}\,dx$. Note that when $x = 2$ we have we have $u = \ln 2$, and as $x \to \infty$, we also have $u = \ln x \to \infty$. Making this substitution, we have

$$\begin{aligned}
\int_2^\infty \frac{1}{x(\ln x)^{3/2}} &= \int_{\ln 2}^\infty \frac{du}{u^{3/2}} \\
&= \lim_{N\to\infty} \int_{\ln 2}^N \frac{du}{u^{3/2}} \\
&= \lim_{N\to\infty} \left[-2u^{-1/2}\right]_{\ln 2}^N \\
&= \lim_{N\to\infty} \left(-\frac{2}{N^{1/2}} + \frac{2}{(\ln 2)^{1/2}}\right) \\
&= \frac{2}{(\ln 2)^{1/2}}.
\end{aligned}$$

□

Problem 2. (3 points each) Determine whether the following series converge or diverge. State why or why not. If the series converges, compute the sum.

(a) $\displaystyle\sum_{n=0}^\infty \left(\frac{3}{2}\right)^n$ (b) $\displaystyle\sum_{n=1}^\infty \frac{1}{2^{n+1}}$

Solution. (a) This diverges since $r = \frac{3}{2} > 1$.

(b) This converges since $r = \frac{1}{2} < 1$. Using the given formula,

$$\begin{aligned}
\sum_{n=1}^\infty \frac{1}{2^{n+1}} &= \sum_{n=0}^\infty \frac{1}{2^{n+2}} \\
&= \sum_{n=0}^\infty \frac{1}{4}\frac{1}{2^n} \\
&= \frac{1}{4}\cdot\frac{1}{1-\frac{1}{2}} \\
&= \frac{1}{4}\cdot 2 \\
&= \frac{1}{2}.
\end{aligned}$$

□