Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (5 points) For the given function and values of z

$$f(x, y) = \ln\left(y - e^{5x}\right), \quad z = 0, \ln 10,$$

(a) What is the domain of this function?

(b) What type of function describes the level curves?

(c) Give a sketch of the level curves.

(d) What functions $y = f(x)$ do you get for these values of z?

Solution. (a) The argument of the logarithm must be positive, so we need $y - e^{5x} > 0$, or $y > e^{5x}$. In set notation, this is

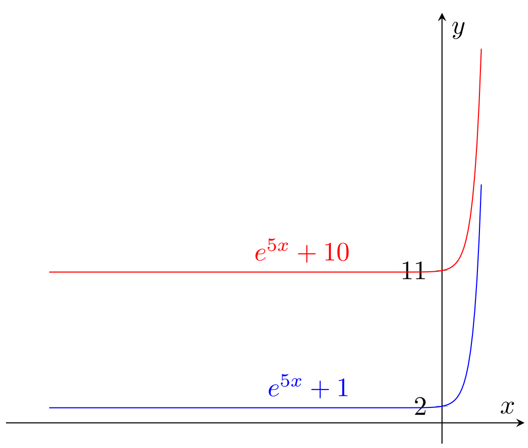
$$\{(x, y) \colon y > e^{5x}\}.$$

(b) The level curves are given by $f(x, y) = k$, where k is a constant. That is,

$$\begin{aligned} k &= \ln(y - e^{5x}) \\ e^k &= y - e^{5x} \\ y &= e^{5x} + e^k, \end{aligned} \tag{$*$}$$

which are exponential functions.

(c) For the given values of $z = 0, \ln 10$ the level curves look like



(d) For the given values of z, we just plug in $k = 0$ and $k = \ln 10$ into $(*)$ to get the functions

$$y = e^{5x} + 1 \quad \text{and} \quad y = e^{5x} + 10.$$

Note that we used the fact that $e^0 = 1$ and $e^{\ln 10} = 10$. □

Problem 2. (4 points) How much money should you invest today at an annual interest rate of 6.7% compounded continuously so that, starting 2 years from today you can make annual withdrawals of \$2600 in perpetuity?

Solution. Recall the continuous compounding interest formula, $A = Pe^{rt}$. Let's think about how much principal we would need each year to be able to withdraw the money we want. If we have exactly the right amount of principal, two years from now, we need to satisfy

$$2600 = P_2 e^{.067(2)},$$

where we are using P_2 to denote the principal at year 2. Solving for P_2, we get

$$P_2 = 2600e^{-.067(2)}.$$

Following the same steps, for year 3, we get

$$P_3 = 2600e^{-.067(3)}.$$

And at any year n, we need

$$P_n = 2600e^{-.067(n)}.$$

So then our initial investment should be all of these things added together to guarantee that we have enough money every year, $P = P_2 + P_3 + P_4 + \cdots$. That is,

$$\begin{aligned}
P &= 2600e^{-.067(2)} + 2600e^{-.067(3)} + 2600e^{-.067(4)} + \cdots \\
&= 2600e^{-.067(2)} \left(1 + e^{-.067} + e^{-.067(2)} + \cdots\right) \\
&= 2600e^{-.067(2)} \sum_{n=0}^{\infty} e^{-.067(n)} \\
&= 2600e^{-.067(2)} \sum_{n=0}^{\infty} \left(e^{-.067}\right)^n \\
&= 2600e^{-.067(2)} \cdot \frac{1}{1 - e^{-.067}} \\
&\approx 35088.98. \square
\end{aligned}$$

Problem 3. (1 points) Which lesson numbers will be relevant for our exam over lessons 13 through 19?

Solution. Lessons 13 through 19. □