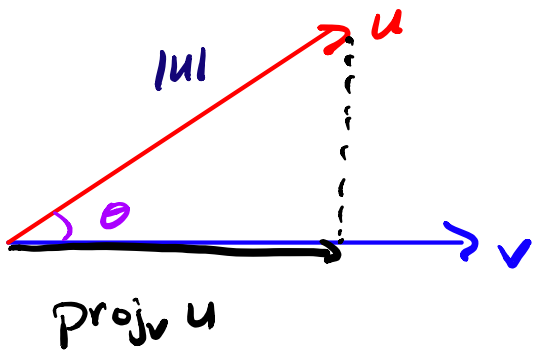
Orthogonal projection

$0 \le \theta < \frac{\pi}{2}$



$$\text{proj}_v u = |u| \cos\theta \, \frac{v}{|v|} = \frac{|u||v|\cos\theta}{|v|^2} v$$

$$= \left(\frac{u \cdot v}{v \cdot v}\right) v$$

$$\text{scal}_v u = |u| \cos\theta \quad \text{"signed length"}$$

$$= \frac{|u||v|\cos\theta}{|v|} = \frac{u \cdot v}{|v|}$$

Example $u = \langle -4, 0, 2 \rangle$, $v = \langle 1, 4, -4 \rangle$

$$\text{proj}_v u = \left(\frac{u \cdot v}{v \cdot v}\right) v = \left(\frac{-4 + 0 - 8}{1 + 16 + 16}\right) \langle 1, 4, -4 \rangle$$

$$= \frac{-12}{33} \langle 1, 4, -4 \rangle$$

$$\text{scal}_v u = \frac{u \cdot v}{|v|} = \frac{-12}{\sqrt{33}}$$

Cross product

Example $u = \langle 1, 2, 3 \rangle$, $v = \langle 4, 5, 6 \rangle$

Method 1

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \hat{k}$$

$$= (12-15)\hat{i} - (6-12)\hat{j} + (5-8)\hat{k}$$

$$= \langle -3, 6, -3 \rangle$$

Method 2

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 12-15 \\ -(6-12) \\ 5-8 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$$