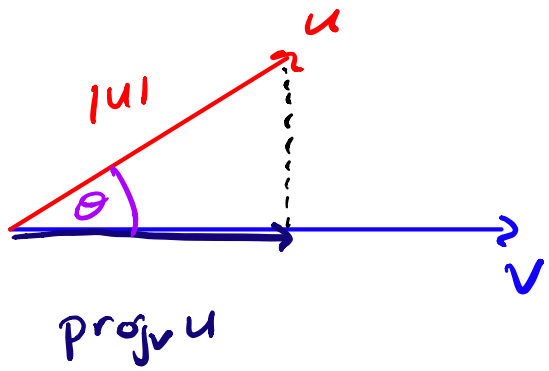
Orthogonal projection

$0 \le \theta < \frac{\pi}{2}$



$$\text{proj}_v u = |u| \cos\theta \; \frac{v}{|v|}$$

$$= \frac{|u||v| \cos\theta \; v}{|v|^2} = \left(\frac{u \cdot v}{v \cdot v}\right) v$$

$$\text{scal}_v u = |u| \cos\theta = \frac{u \cdot v}{|v|}$$

"signed length"

Ex $u = \langle -4, 0, 2 \rangle$, $v = \langle 1, 4, -4 \rangle$

$$\text{proj}_v u = \left(\frac{u \cdot v}{v \cdot v}\right) v = \left(\frac{-4 + 0 - 8}{1 + 16 + 16}\right) \langle 1, 4, -4 \rangle$$

$$= \frac{-12}{33} \langle 1, 4, -4 \rangle$$

$|v|^2$

$$\text{scal}_v u = \frac{-12}{\sqrt{33}}$$

Cross product

$u = \langle 1, 2, 3 \rangle, \quad v = \langle 4, 5, 6 \rangle$

Method 1

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \hat{k}$$

$$= (12 - 15)\hat{i} - (6 - 12)\hat{j} + (5 - 8)\hat{k}$$

$$= \langle -3, 6, -3 \rangle$$

Method 2

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 - 15 \\ -(6 - 12) \\ 5 - 8 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$$

13.3.57

u, $\hat{k}$, $\theta = 60°$

unit vector

$$\text{proj}_k u = |u| \cos\theta \, \frac{k}{|k|}$$

$$= |u| \cdot \frac{1}{2} \cdot \langle 0, 0, 1 \rangle$$

$$= \langle 0, 0, \tfrac{1}{2} \rangle$$