13.6.54

$$x^2 + y^2 + 7z^2 + 12x = -35$$

$$x^2 + 12x + 36 + y^2 + 7z^2 = -35 + 36$$

$$(x+6)^2 + y^2 + \frac{z^2}{(1/\sqrt{7})^2} = 1$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1}$$

Ellipsoid

Center: $(-6, 0, 0)$

$|x+6| \le 1$

$-7 \le x \le -5$

length in x-direction: 2

y-direction: 2

z-direction: $2/\sqrt{7}$

13.5.33

l_1: $x = 2,\ y = 7 - t,\ z = 1 + t$

l_2: $x = -3 - 5s,\ y = 1 + 3s,\ z = 8 - s$

intersect $\rightarrow$ coordinates equal

$2 = -3 - 5s$

$\boxed{s = -1}$

$\vec{r}_2(-1) = (2, -2, 9)$

Also need $y = 7 - t = -2 \rightarrow t = 9$

$z = 1 + t = 9 \rightarrow t = 8$

No solution for $t \rightarrow$ don't intersect

l_1 : direction vector $\langle 0, -1, 1 \rangle = v_1$

l_2 $\langle -5, 3, -1 \rangle = v_2$

v_2 not multiple of $v_1 \rightarrow$ not parallel $\rightarrow$ skew

Week 1

- Dot product
  - orthogonal projection
- Cross product
  - area of parallelogram
- Eqn of line and plane
- ellipsoid, cylinder, elliptic paraboloid

$\vec{u}$, $\vec{v}$

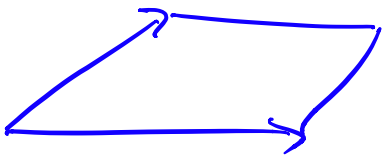
$$\text{proj}_{\vec{v}}\,\vec{u} = \left(\frac{u \cdot v}{v \cdot v}\right) v$$

$\vec{u} = \langle 1, 2, 3 \rangle$, $\vec{v} = \langle -1, -2, 4 \rangle$

$$\text{proj}_{\vec{v}}\,\vec{u} = \frac{-1 - 4 + 12}{1 + 4 + 16} \langle -1, -2, 4 \rangle$$

Parallelogram spanned by $\vec{v}, \vec{w}$

$$\text{Area} = |v \times w| = |v||w| \sin\theta$$



Eqn of a line

$$\vec{r}(t) = \vec{r}_0 + t\,\vec{v}$$

$\vec{r}_0$: initial

$\vec{v}$: direction

Eqn of plane

point, normal vector

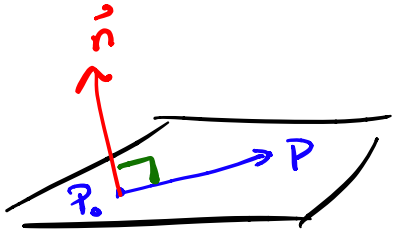
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$\vec{n} = \langle a, b, c \rangle$

$P_0 = (x_0, y_0, z_0)$

$P = (x, y, z)$

$\overrightarrow{P_0P} = \langle x - x_0, y - y_0, z - z_0 \rangle$



What is a cylinder?

"sheet defined by any plane curve"