13.6.54

$$x^2 + y^2 + 7z^2 + 12x = -35$$

$$x^2 + 12x + 36 + y^2 + 7z^2 = -35 + 36$$

$$(x+6)^2 + y^2 + 7z^2 = 1$$

$$(x+6)^2 + y^2 + \frac{z^2}{(1/\sqrt{7})^2} = 1$$

Ellipsoid. Center: $(-6, 0, 0)$

Length in x-direction: 2

y-direction: 2

z-direction: $2/\sqrt{7}$

$$(x+6)^2 \leq 1$$

$$|x+6| \leq 1$$

$$-1 \leq x+6 \leq 1$$

$$-7 \leq x \leq -5$$

13.5. 73

$Q: \; -x-3y+2z=1 \; ; \; R: \; x+y+z=0$

Find line of intersection

Algebra

$$\begin{array}{r} -x-3y+2z=1 \\ x+y+z=0 \\ \hline -2y+3z=1 \end{array}$$

$$3z = 1+2y$$

$$\boxed{z = \frac{1}{3}+\frac{2}{3}y}$$

$$x+y+\frac{1}{3}+\frac{2}{3}y=0$$

$$x+\frac{1}{3}+\frac{5}{3}y=0$$

$$\boxed{x = -\frac{1}{3}-\frac{5}{3}y}$$

Pick $y=t$

Calc way

$n_1 = \langle -1, -3, 2 \rangle, \quad n_2 = \langle 1, 1, 1 \rangle$

$$n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ 1 & 1 & 1 \end{vmatrix} = (-3-2)\hat{i} - (-1-2)\hat{j} + (-1+3)\hat{k}$$

$$\boxed{\vec{v} = \langle -5, 3, 2 \rangle}$$

Need a point

Arbitrarily pick $z = 0$

$$\begin{aligned} -x - 3y &= 1 \\ x + y &= 0 \\ \hline -2y &= 1 \end{aligned} \rightarrow \boxed{y = -\tfrac{1}{2}}$$

$$\downarrow$$

$$\boxed{x = \tfrac{1}{2}}$$

$$\boxed{P = \left(\tfrac{1}{2}, -\tfrac{1}{2}, 0\right)}$$

$$\vec{r}(t) = \langle \tfrac{1}{2}, -\tfrac{1}{2}, 0 \rangle + t\langle -5, 3, 2 \rangle$$

13.5 Eqn of line

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$

initial

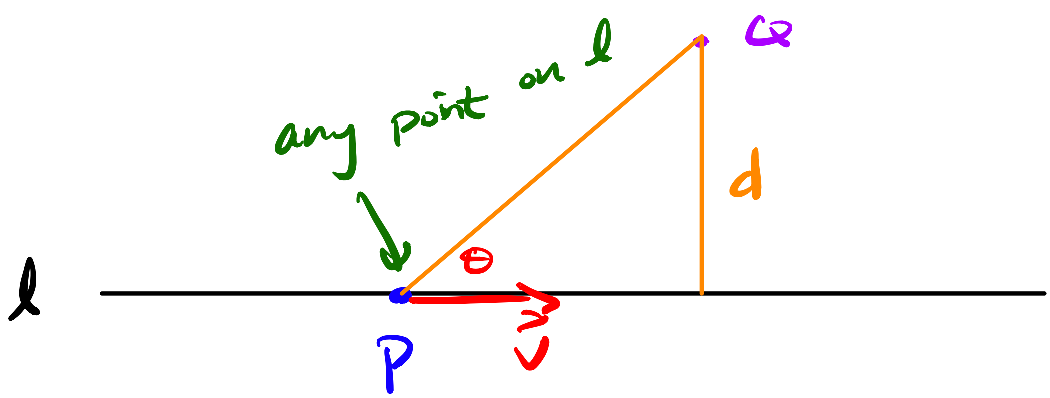
direction

Think in terms of this form

Keep in mind $r(t) = \langle x(t), y(t), z(t) \rangle$

Given Q, a line ℓ: $r(t) = \vec{r}_0 + t\vec{v}$

Find distance d from Q to ℓ.



$|PQ| \sin\theta = d$

$|\vec{v} \times \vec{PQ}| = |\vec{v}|\,|\vec{PQ}| \sin\theta$ by def

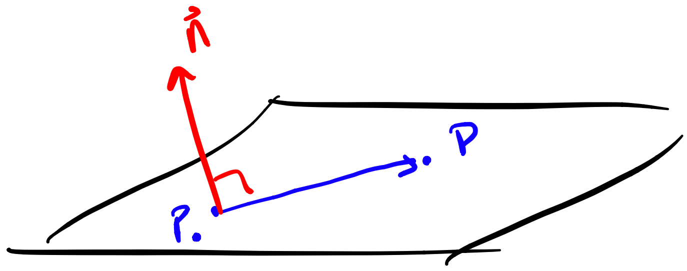
$= |v|\, d \longrightarrow d = \dfrac{|\vec{v} \times \vec{PQ}|}{|\vec{v}|}$

Eqn of plane

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$\vec{n} = \langle a, b, c \rangle$

$n \cdot \overrightarrow{P_0P}$



Plane defined by point and normal vector

line defined by point and direction vector