*13.4.7

*13.4.33

15.2.34

*13.4.7

$$z = -5e^x \ln y, \quad x = \ln(u\cos v), \quad y = u\sin v$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial x} = -5e^x \ln y \qquad \frac{\partial x}{\partial u} = \frac{1}{u} \left(= \frac{\cos v}{u \cdot \cos v} \right)$$

$$\frac{\partial z}{\partial y} = \frac{-5e^x}{y} \qquad \frac{\partial y}{\partial u} = \sin v$$

$$\frac{\partial z}{\partial u} = -5e^x \ln y \cdot \frac{1}{u} - \frac{5e^x}{y}\sin v$$

*13.4.33 (See note at end)

$$9\sin(x+y) + 4\sin(x+z) + 9\sin(y+z) = 0$$

$$\frac{\partial z}{\partial y}\Bigg|\; 9\cos(x+y) + 4\cos(x+z)\cdot\frac{\partial z}{\partial y} + 9\cos(y+z)\frac{\partial z}{\partial y} = 0$$

Evaluate at $(x,y,z) = (4\pi, 3\pi, 3\pi)$

$$9\cos(4\pi+3\pi) + 4\cos(4\pi+3\pi)\frac{\partial z}{\partial y} + 9\cos(3\pi+3\pi)\frac{\partial z}{\partial y} = 0$$

$$-9 - 4\frac{\partial z}{\partial y} + 9\frac{\partial z}{\partial y} = 0$$

$$\boxed{\frac{\partial z}{\partial y} = 9/5}$$

15.2.34

$$f(x,y) = \frac{-x}{\sqrt{x^2+y^2}}$$

Show limit DNE $(x,y) \to (0,0)$

Along $y=x$, $x>0$

$$f(x,x) = \frac{-x}{\sqrt{2x^2}} = \frac{-x}{\sqrt{2}\,|x|}$$

$$\underline{x>0} \quad = \frac{-1}{\sqrt{2}} \xrightarrow[x\to 0]{} \frac{-1}{\sqrt{2}}$$

Along $y=x$, $x<0$

$$f(x,x) = \frac{1}{\sqrt{2}} \xrightarrow[x\to 0]{} \frac{1}{\sqrt{2}}$$

$\Rightarrow$ limit DNE (depends on path)

$$f(x,y) = \sqrt{4 - x^2 - y^2} = \sqrt{4 - (x^2 + y^2)}$$

<u>range</u> $0 \leq z \leq 2$

<u>domain</u> $x^2 + y^2 \leq 4$

$$f(x,y) = 2 - \sqrt{4 - x^2 - y^2}$$

multiply original range by -1 then add 2

$$2 - \sqrt{}$$

$$-2 \leq z \leq 0$$
$$+2 \qquad\qquad +2$$

$$0 \leq z \leq 2$$

An alternative way of computing $\frac{\partial z}{\partial y}$ using implicit differentiation:

Given $F(x,y,z) = c$ (c is some constant)

$$0 = \frac{\partial}{\partial y} F(x,y,z) = F_y \frac{\partial y}{\partial y} + F_z \cdot \frac{\partial z}{\partial y}$$

$$= F_y + F_z \frac{\partial z}{\partial y}$$

$$\Rightarrow \frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$$

Similarly for $\frac{\partial z}{\partial x}$.